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Trigonometry

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[1] Each lesson is ONE day, and ONE day is considered a 45-minute period.

Precalculus and Advanced Topics • Module 4

Trigonometry

OVERVIEW

Trigonometry was introduced in Geometry through a study of right triangles. In Algebra II, work was conducted on extending basic trigonometry to the domain of all real numbers via the unit circle. This module revisits, unites, and expands further those ideas and introduces new tools for solving geometric and modeling problems through the power of trigonometry.

Topic A helps students recall how to use special triangles positioned within the unit circle to determine geometrically the values of sine, cosine, and tangent at special angles. The unit circle is then used to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number (**F-TF.A.3**) and to explain the periodicity of the trigonometric functions and their symmetries (**F-TF.A.4**). Students develop the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems (**F-TF.C.9**) and to model geometric phenomena. Students also discuss the construction of tangent lines to circles (**G-C.A.4**) and revisit the geometric origins of the tangent function. Student exploration of tangents through a paper folding and compass activity is then used in modeling waves, waves traveling together, and wave patterns generated by musical instruments.

Students derive sophisticated applications of the trigonometric functions in Topic B including: the area formula for a general triangle, $A = \left(\frac{1}{2}\right) ab \sin(\theta)$ (**G-SRT.D.9**), the law of sines, the law of cosines, and Heron's formula. They use previous knowledge and apply their understanding of the Pythagorean theorem and oblique triangles to discover these formulas while analyzing patterns. Finally, as students investigate force diagrams and paths across rivers, they solve survey and elevation problems and revisit vectors (**G-SRT.D.10**, **G-SRT.D.11**).

The graphs of the trigonometric functions are revisited in Topic C. Students visualize these graphs with the aid of the appropriate software and briefly recall how changing various parameters of a trigonometric function affects its graph. Students extend their knowledge of inverse functions to trigonometric functions (**F-TF.B.6**) as they restrict domains to create inverse trigonometric functions. These inverse functions are then used to solve trigonometric equations, evaluate their solutions using technology, and interpret these solutions in the appropriate contexts (**F-TF.B.7**). Students determine viewing angle, line of sight, height of objects, and the angle of elevation for inclined surfaces using inverse trigonometric functions and their periodic phenomena.

Focus Standards

Extend the domain of trigonometric functions using the unit circle.

F-TF.A.3   (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$, and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

F-TF.A.4   (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions.

F-TF.B.6   (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

F-TF.B.7   (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.[★]

Prove and apply trigonometric identities.

F-TF.C.9[2]   (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Understand and apply theorems about circles.

G-C.A.4   (+) Construct a tangent line from a point outside a given circle to the circle.

Apply trigonometry to general triangles.

G-SRT.D.9   (+) Derive the formula $A = 1/2 \, ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

G-SRT.D.10   (+) Prove the Laws of Sines and Cosines and use them to solve problems.

G-SRT.D.11   (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Foundational Standards

Extend the domain of trigonometric functions using the unit circle.

F-TF.A.1   Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

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[2] Students are now responsible for proofs of angle addition and subtraction formulas.

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F-TF.A.2   Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Model periodic phenomena with trigonometric functions.

F-TF.B.5   Choose trigonometry functions to model periodic phenomena with specified amplitude, frequency, and midline.★

Prove and apply trigonometric identities.

F-TF.C.8   Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

Understand the concept of a function and use function notation.

F-IF.A.1   Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

F-IF.A.2   Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Analyze functions using different representations.

F-IF.C.7   Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

        e.   Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Build new functions from existing functions.

F-BF.B.4   Find inverse functions.

        a.   Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ or $f(x) = x + 1/x - 1$ for $x \neq 1$.*

        b.   (+) Verify by composition that one function is the inverse of another.

        c.   (+) Read values of an inverse function from a graph or table, given that the function has an inverse.

        d.   (+) Produce an invertible function from a non-invertible function by restricting the domain.

Define trigonometric ratios and solve problems involving right triangles.

G-SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G-SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.

G-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★

Understand and apply theorems about circles.

G-C.A.2 Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

Prove geometric theorems.

G-CO.C.10 Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to* $180°$*; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

Focus Standards for Mathematical Practice

MP.3 **Construct viable arguments and critique the reasoning of others.** Students construct mathematical arguments as they explain their calculations of the area of triangles leading to a new formula for area, $\text{Area} = \frac{1}{2}bc\sin(\theta)$. Students explain the properties of trigonometric functions and explain construction of tangent lines. Students use the periodic nature of trigonometric functions to reason about their graphs and problem solve using inverse functions.

MP.4 **Model with mathematics**. Students apply sum and difference formulas in the context of modeling sound waves using trigonometric functions. Students model using trigonometric functions applying tangent lines and Law of Sines and Cosines to solve surveying problems and revisit vectors. Students investigate viewing distance, line of sight, and viewing angle using inverse trigonometric functions as well as the angle of elevation for inclined surfaces.

MP.5 **Use appropriate tools strategically**. Students see the unit circle as a tool to determine the values of trigonometric functions in terms of x and explain their periodicity and symmetry. Students use computer software and graphing calculators to graph trigonometric functions and their inverses. Students see trigonometric inverses, Law of Sines, Law of Cosines, and area formulas as tools in problem solving.

Terminology

Familiar Terms and Symbols[3]

- Sine
- Cosine
- Tangent
- Function
- Radian Measure
- Tangent Line
- Unit Circle
- Period
- Vector

Suggested Tools and Representations

- Compass
- Straightedge
- Graphing Calculator
- Wolfram Alpha Software
- Geometer's Sketchpad Software
- GeoGebra software

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic A	Constructed response with rubric	F-TF.A.3, F-TF.A.4, F-TF.C.9, G-C.A.4
End-of-Module Assessment Task	After Topic C	Constructed response with rubric	F-TF.B.6, F-TF.B.7, G-SRT.D.9, G-SRT.D.10, G-SRT.D.11

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[3] These are terms and symbols students have seen previously.