Lesson 28: When Can We Reverse a Transformation?

Classwork

Opening Exercise

Perform the operation $\begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$ on the unit square.

a. State the vertices of the transformation.

b. Explain the transformation in words.

c. Find the area of the transformed figure.

d. If the original square was 2×2 instead of a unit square, how would the transformation change?

e. What is the area of the image? Explain how you know.

Example 1

What transformation reverses a pure dilation from the origin with a scale factor of k?

a. Write the pure dilation matrix and multiply it by $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$.

b. What values of a, b, c, and d would produce the identity matrix? (Hint: Write and solve a system of equations.)

c. Write the matrix and confirm that it reverses the pure dilation with a scale factor of k.

Exercises 1–3

Find the inverse matrix and verify.

1. $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

3. $\begin{bmatrix} -2 & -5 \\ 1 & 2 \end{bmatrix}$

Problem Set

1. In this lesson, we learned $R_\theta R_{-\theta} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Chad was saying that he found an easy way to find the inverse matrix, which is: $R_{-\theta} = \frac{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}{R_\theta}$. His argument is that if we have $2x = 1$, then $x = \frac{1}{2}$.
   a. Is Chad correct? Explain your reason.
   b. If Chad is not correct, what is the correct way to find the inverse matrix?

2. Find the inverse matrix and verify it.
   a. $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$
   b. $\begin{bmatrix} -2 & -1 \\ 3 & 1 \end{bmatrix}$
   c. $\begin{bmatrix} 3 & -3 \\ -2 & 2 \end{bmatrix}$
   d. $\begin{bmatrix} 0 & 1 \\ -1 & 3 \end{bmatrix}$
   e. $\begin{bmatrix} 4 & 1 \\ 2 & 1 \end{bmatrix}$

3. Find the starting point $\begin{bmatrix} x \\ y \end{bmatrix}$ if
   a. the point $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ is the image of a pure dilation with a factor of 2.
   b. the point $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ is the image of a pure dilation with a factor of $\frac{1}{2}$.
   c. the point $\begin{bmatrix} -10 \\ 35 \end{bmatrix}$ is the image of a pure dilation with a factor of 5.
   d. the point $\begin{bmatrix} \frac{4}{9} \\ \frac{16}{21} \end{bmatrix}$ is the image of a pure dilation with a factor of $\frac{2}{3}$.

4. Find the starting point if
   a. $3 + 2i$ is the image of a reflection about the real axis.
   b. $3 + 2i$ is the image of a reflection about the imaginary axis.
   c. $3 + 2i$ is the image of a reflection about the real axis and then the imaginary axis.
   d. $-3 - 2i$ is the image of a π radians counterclockwise rotation.

5. Let's call the pure counterclockwise rotation of the matrix $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ as R_θ, and the "undo" of the pure rotation is $\begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix}$ as $R_{-\theta}$.
   a. Simplify $\begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix}$.

b. What would you get if you multiply R_θ to $R_{-\theta}$?

c. Write the matrix if you want to rotate $\frac{\pi}{2}$ radians counterclockwise.

d. Write the matrix if you want to rotate $\frac{\pi}{2}$ radians clockwise.

e. Write the matrix if you want to rotate $\frac{\pi}{6}$ radians counterclockwise.

f. Write the matrix if you want to rotate $\frac{\pi}{4}$ radians counterclockwise.

g. If the point $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ is the image of $\frac{\pi}{4}$ radians counterclockwise rotation, find the starting point $\begin{bmatrix} x \\ y \end{bmatrix}$.

h. If the point $\begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix}$ is the image of $\frac{\pi}{6}$ radians counterclockwise rotation, find the starting point $\begin{bmatrix} x \\ y \end{bmatrix}$.