Lesson 28: When Can We Reverse a Transformation?

Classwork

Opening Exercise

Perform the operation $\left[\begin{matrix}3&-2\\1&1\end{matrix}\right]$ on the unit square.

* 1. State the vertices of the transformation.
	2. Explain the transformation in words.
	3. Find the area of the transformed figure.
	4. If the original square was $2×2$ instead of a unit square, how would the transformation change?
	5. What is the area of the image? Explain how you know.

**Example 1**

What transformation reverses a pure dilation from the origin with a scale factor of $k$?

* 1. Write the pure dilation matrix and multiply it by $\left[\begin{matrix}a&c\\b&d\end{matrix}\right]$.
	2. What values of $a, b, c$, and $d$ would produce the identity matrix? (Hint: Write and solve a system of equations.)
	3. Write the matrix and confirm that it reverses the pure dilation with a scale factor of $k$.

Exercises 1–3

Find the inverse matrix and verify.

1. $\left[\begin{matrix}1&0\\1&1\end{matrix}\right]$
2. $\left[\begin{matrix}3&1\\5&2\end{matrix}\right]$
3. $\left[\begin{matrix}-2&-5\\1&2\end{matrix}\right]$

Problem Set

1. In this lesson, we learned $R\_{θ}R\_{-θ}=\left[\begin{matrix}1&0\\0&1\end{matrix}\right]$. Chad was saying that he found an easy way to find the inverse matrix, which is: $R\_{-θ}=\frac{ \left[\begin{matrix}1&0\\0&1\end{matrix}\right] }{R\_{θ}}.$ His argument is that if we have $2x=1$, then $x=\frac{ 1 }{ 2 }$ .
	1. Is Chad correct? Explain your reason.
	2. If Chad is not correct, what is the correct way to find the inverse matrix?
2. Find the inverse matrix and verify it.
	1. $\left[\begin{matrix}3&2\\7&5\end{matrix}\right]$
	2. $\left[\begin{matrix}-2&-1\\3&1\end{matrix}\right]$
	3. $\left[\begin{matrix}3&-3\\-2&2\end{matrix}\right]$
	4. $\left[\begin{matrix}0&1\\-1&3\end{matrix}\right]$
	5. $\left[\begin{matrix}4&1\\2&1\end{matrix}\right]$
3. Find the starting point $\left[\begin{matrix}x\\y\end{matrix}\right]$ if
	1. the point $\left[\begin{matrix}4\\2\end{matrix}\right]$ is the image of a pure dilation with a factor of $2$.
	2. the point $\left[\begin{matrix}4\\2\end{matrix}\right]$ is the image of a pure dilation with a factor of $\frac{ 1 }{ 2 }$.
	3. the point $\left[\begin{matrix}-10\\35\end{matrix}\right]$ is the image of a pure dilation with a factor of $5$.
	4. the point $\left[\begin{matrix}\frac{ 4 }{ 9 }\\\frac{ 16 }{ 21 }\end{matrix}\right]$ is the image of a pure dilation with a factor of $\frac{ 2 }{ 3 }$.
4. Find the starting point if
	1. $3+2i$ is the image of a reflection about the real axis.
	2. $3+2i$ is the image of a reflection about the imaginary axis.$ $
	3. $3+2i$ is the image of a reflection about the real axis and then the imaginary axis.
	4. $-3-2i$ is the image of a $π$ radians counterclockwise rotation.
5. Let’s call the pure counterclockwise rotation of the matrix $\left[\begin{matrix}cos θ&-sin θ\\sin θ&cos θ\end{matrix}\right]$ as $R\_{θ} $, and the “undo” of the pure rotation is $\left[\begin{matrix}cos (-θ)&-sin (-θ)\\sin (-θ)&cos (-θ)\end{matrix}\right]$ as $R\_{-θ}$ .
	1. Simplify $\left[\begin{matrix}cos (-θ)&-sin (-θ)\\sin (-θ)&cos (-θ)\end{matrix}\right]$ .
	2. What would you get if you multiply $R\_{θ} $ to $R\_{-θ}$ ?
	3. Write the matrix if you want to rotate $\frac{ π }{ 2 }$ radians counterclockwise.
	4. Write the matrix if you want to rotate $\frac{ π }{ 2 }$ radians clockwise.
	5. Write the matrix if you want to rotate $\frac{ π }{ 6 }$ radians counterclockwise.
	6. Write the matrix if you want to rotate $\frac{ π }{ 4 }$ radians counterclockwise.
	7. If the point $\left[\begin{matrix}0\\2\end{matrix}\right]$ is the image of $\frac{ π }{ 4 }$ radians counterclockwise rotation, find the starting point $\left[\begin{matrix}x\\y\end{matrix}\right]$.
	8. If the point $\left[\begin{matrix}\frac{ 1 }{ 2 }\\\frac{ \sqrt{3} }{ 2}\end{matrix}\right]$ is the image of $\frac{ π }{ 6 }$ radians counterclockwise rotation, find the starting point $\left[\begin{matrix}x\\y\end{matrix}\right]$.