Lesson 26: Getting a Handle on New Transformations

Classwork

Opening Exercise

Perform the following matrix operations:

a. $\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

b. $\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$

d. $\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

e. $\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

f. $\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$

g. $\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

h. Can you add the two matrices in part (a)? Why or why not?

Exercises 1–3

1. Perform the transformation $\begin{bmatrix} 109 & 3 \\ 1 & -2 \end{bmatrix}$ on the unit square.

   a. Sketch the image. What is the shape of the image?

   b. What are the coordinates of the vertices of the image?

   c. What is the area of the image? Show your work.

2. In the Exploratory Challenge, we drew the image of a general rotation/dilation of the unit square $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$.

   a. Calculate the area of the image by enclosing the image in a rectangle and subtracting the area of surrounding right triangles. Show your work.

   b. Confirm the area using the determinant of the resulting matrix.

3. We have looked at several general matrix transformations in Module 1. Answer the questions below about these familiar matrices and explain your answers.

   a. What effect does the identity transformation have on the unit square? What is the area of the image? Confirm your answer using the determinant.

   b. How does a dilation with a scale factor of k change the area of the unit square? Calculate the determinant of a matrix representing a pure dilation of k.

   c. Does a rotation with no dilation change the area of the unit square? Confirm your answer by calculating the determinant of a pure rotation matrix and explain.

Lesson Summary

Definition

- The area of the image of the unit square under the linear transformation represented by a 2×2 matrix is called the *determinant* of that matrix.

Problem Set

1. Perform the following transformation on the unit square: sketch the image and the area of the image.
   a. $\begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix}$
   b. $\begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix}$
   c. $\begin{bmatrix} 4 & -2 \\ 2 & 4 \end{bmatrix}$
   d. $\begin{bmatrix} 2 & -4 \\ 4 & 2 \end{bmatrix}$

2. Perform the following transformation on the unit square: sketch the image, find the determinant of the given matrix, and find the area the image.
   a. $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$
   b. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
   c. $\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$
   d. $\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$
   e. The determinants in parts (a), (b), (c), and (d) have positive or negative values. What is the value of the determinants if the vertices (b, c) and (c, d) are switched?

3. Perform the following transformation on the unit square: sketch the image, find the determinant of the given matrix, and find the area the image.
   a. $\begin{bmatrix} -1 & -3 \\ -2 & -4 \end{bmatrix}$
   b. $\begin{bmatrix} -1 & -3 \\ 2 & 4 \end{bmatrix}$
   c. $\begin{bmatrix} 1 & 3 \\ -2 & -4 \end{bmatrix}$
   d. $\begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$
   e. $\begin{bmatrix} 1 & -3 \\ 2 & -4 \end{bmatrix}$

f. $\begin{bmatrix} -1 & 3 \\ 2 & -4 \end{bmatrix}$

g. $\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix}$