Lesson 21: The Hunt for Better Notation

Classwork

Opening Exercise

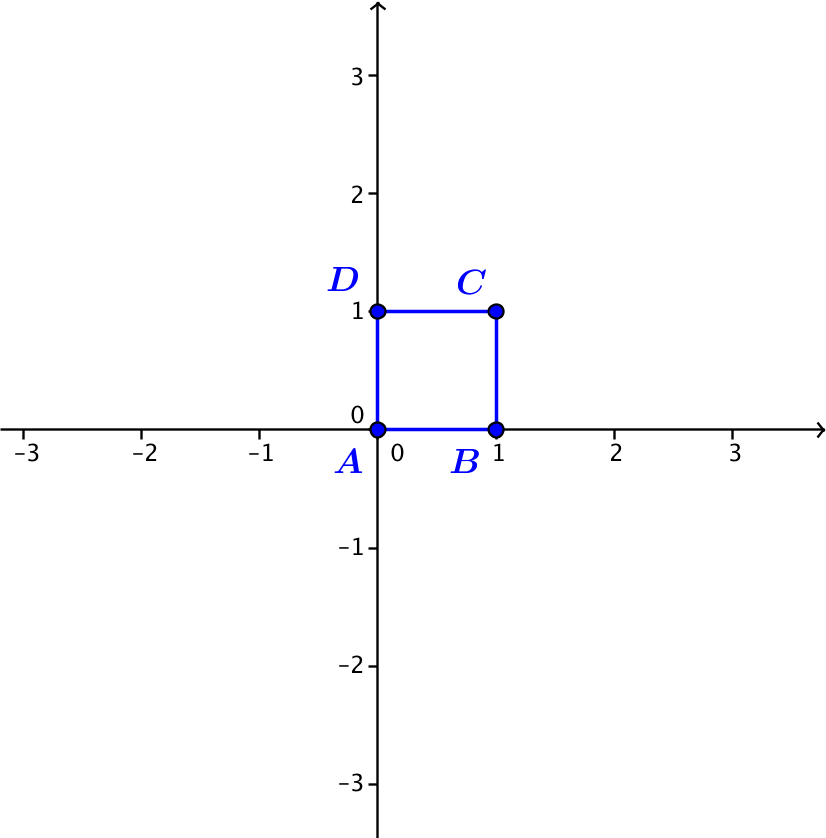
Suppose that and .  
Find the result of performing and then on a point . That is, find .

Exercises 1–2

1. Calculate each of the following products.
2. Find a value of so that .

Exercises 3–9

1. Find a matrix so that we can represent the transformation by   
   .
2. If a transformation has the geometric effect of rotation and dilation, do you know about the values , and ?
3. Describe the form of a matrix so that the transformation has the geometric effect of only dilation by a scale factor .
4. Describe the form of a matrix so that the transformation has the geometric effect of only rotation by . Describe the matrix in terms of .
5. Describe the form of a matrix so that the transformation has the geometric effect of rotation by and dilation with scale factor . Describe the matrix in terms of and .
6. Suppose that we have a transformation .
   1. Does this transformation have the geometric effect of rotation and dilation?
   2. Transform each of the points , , and and plot the images in the plane shown.



1. Describe the geometric effect of the transformation .

Lesson Summary

For real numbers , and , the transformation can be represented using matrix multiplication by , where and the represents the point in the plane.

* The transformation is a counterclockwise rotation by if and only if the matrix representation is  
  .
* The transformation is a dilation with scale factor if and only if the matrix representation is   
  .
* The transformation is a counterclockwise rotation by and dilation with scale factor if and only if the matrix representation is . If we let and   
  , then the matrix representation is .

Problem Set

1. Perform the indicated multiplication.
2. Find a value of so that .
3. Find values of and so that .
4. Find values of and so that .
5. Write the following transformations using matrix multiplication.
6. Identify whether or not the following transformations have the geometric effect of rotation only, dilation only, rotation and dilation only, or none of these.
7. Create a matrix representation of a linear transformation that has the specified geometric effect.
   1. Dilation by a factor of and no rotation.
   2. Rotation by and no dilation.
   3. Rotation by and dilation by a scale factor of .
   4. Rotation by and dilation by a scale factor of .
8. Identify the geometric effect of the following transformations. Justify your answer.