Lesson 20: Exploiting the Connection to Cartesian Coordinates

Classwork

Opening Exercise

* 1. Find a complex number $w$ so that the transformation $L\_{1}\left(z\right)=wz$ produces a clockwise rotation by $1°$ about the origin with no dilation.
	2. Find a complex number $w$ so that the transformation $L\_{2}\left(z\right)=wz$ produces a dilation with scale factor $0.1$ with no rotation.

Exercises 1–4

* 1. Find values of $a$ and $b$ so that $L\_{1}\left(x,y\right)=\left(ax-by, bx+ay\right)$ has the effect of dilation with scale factor $2$ and no rotation.
	2. Evaluate $L\_{1}\left(L\_{1}\left(x,y\right)\right)$, and identify the resulting transformation.
	3. Find values of $a$ and $b$ so that $L\_{2}\left(x,y\right)=\left(ax-by, bx+ay\right)$ has the effect of rotation about the origin by $180°$ counterclockwise and no dilation.
	4. Evaluate $L\_{2}\left(L\_{2}\left(x,y\right)\right)$, and identify the resulting transformation.
	5. Find values of $a$ and $b$ so that $L\_{3}\left(x,y\right)=\left(ax-by, bx+ay\right)$ has the effect of rotation about the origin by $90°$ counterclockwise and no dilation.
	6. Evaluate $L\_{3}\left(L\_{3}\left(x,y\right)\right)$, and identify the resulting transformation.
	7. Find values of $a$ and $b$ so that $L\_{3}\left(x,y\right)=\left(ax-by, bx+ay\right)$ has the effect of rotation about the origin by $45°$ counterclockwise and no dilation.
	8. Evaluate $L\_{4}\left(L\_{4}\left(x,y\right)\right)$, and identify the resulting transformation.

Exercises 5–6

1. The figure below shows a quadrilateral with vertices $A\left(0,0\right)$, $B(1,0)$, $C(3,3)$, and $D(0,3)$.
	1. Transform each vertex under $L\_{5}=(3x+y, 3y-x)$, and plot the transformed vertices on the figure.



* 1. Does $L\_{5}$ represent a rotation and dilation? If so, estimate the amount of rotation and the scale factor from your figure.
	2. If $L\_{5}$ represents a rotation and dilation, calculate the amount of rotation and the scale factor from the formula for $L\_{5}$. Do your numbers agree with your estimate in part (b)? If not, explain why there are no values of $a$ and $b$ so that $L\_{5}\left(x,y\right)=\left(ax-by, bx+ay\right)$.
1. The figure below shows a figure with vertices $A\left(0,0\right)$, $B(1,0)$, $C(3,3)$, and $D(0,3)$.
	1. Transform each vertex under $L\_{6}=\left(2x+2y, 2x-2y\right)$, and plot the transformed vertices on the figure.



* 1. Does $L\_{6}$ represent a rotation and dilation? If so, estimate the amount of rotation and the scale factor from your figure.
	2. If $L\_{5}$ represents a rotation and dilation, calculate the amount of rotation and the scale factor from the formula for $L\_{6}$. Do your numbers agree with your estimate in part (b)? If not, explain why there are no values of $a$ and $b$ so that $L\_{6}\left(x,y\right)=\left(ax-by, bx+ay\right)$.

Lesson Summary

For real numbers $a$ and $b$, the transformation $L\left(x,y\right)=(ax-by, bx+ay)$ corresponds to a counterclockwise rotation by $arg(a+bi)$ about the origin and dilation with scale factor $\sqrt{a^{2}+b^{2}}$.

Problem Set

1. Find real numbers $a$ and $b$ so that the transformation $L\left(x,y\right)=\left(ax-by, bx+ay\right)$ produces the specified rotation and dilation.
	1. Rotation by $270°$ counterclockwise and dilation by scale factor $\frac{1}{2}$.
	2. Rotation by $135°$ counterclockwise and dilation by scale factor $\sqrt{2}$.
	3. Rotation by $45°$ clockwise and dilation by scale factor $10$.
	4. Rotation by $540°$ counterclockwise and dilation by scale factor $4$.
2. Determine if the following transformations represent a rotation and dilation. If so, identify the scale factor and the

amount of rotation.



* 1. $L\left(x,y\right)=\left(3x+4y, 4x+3y\right)$
	2. $L\left(x,y\right)=(-5x+12y, -12x-5y)$
	3. $L\left(x,y\right)=(3x+3y, -3y+3x)$
1. Grace and Lily have a different point of view about the transformation on cube $ABCD$ that is shown above. Grace states that it is a reflection about the imaginary axis and a dilation of factor of $2$. However, Lily argues it should be a $90° $counterclockwise rotation about the origin with a dilation of a factor of $2$.
	1. Who is correct? Justify your answer.
	2. Represent the above transformation in the form $L\left(x,y\right)=(ax-by, bx+ay)$.
2. Grace and Lily still have a different point of view on this transformation on triangle $ABC$ shown above. Grace states that it is reflected about the real axis first, then reflected about the imaginary axis, and then is dilated with a factor of $2$. However, Lily asserts that it is a $180°$ counterclockwise rotation about the origin with a dilation of a factor of $2$.



* 1. Who is correct? Justify your answer.
	2. Represent the above transformation in the form $L\left(x,y\right)=(ax-by, bx+ay)$.
1. Given $z=\sqrt{3}+i$.
	1. Find the complex number $w$ that will cause a rotation with the same number of degrees as $z$ without a dilation.
	2. Can you come up with a general formula $L\left(x,y\right)=(ax-by, bx+ay)$ for any complex number $z=x+yi$ to represent this condition?