



Topic C:

The Power of the Right Notation

N-CN.B.4, N-CN.B.5, N-VM.C.8, N-VM.C.10, N-VM.C.11, N-VM.C.12

Focus Standards:	N-CN.B.4	(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
	N-CN.B.5	(+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. <i>For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument 120°.</i>
	N-VM.C.8	(+) Add, subtract, and multiply matrices of appropriate dimensions.
	N-VM.C.10	(+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
	N-VM.C.11	(+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
	N-VM.C.12	(+) Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Instructional Days: 13

- Lessons 18–19:** Exploiting the Connection to Trigonometry (E, P)¹
- Lesson 20:** Exploiting the Connection to Cartesian Coordinates (S)
- Lesson 21:** The Hunt for Better Notation (P)
- Lessons 22–23:** Modeling Video Game Motion with Matrices (P, P)
- Lesson 24:** Matrix Notation Encompasses New Transformations! (P)
- Lesson 25:** Matrix Multiplication and Addition (P)
- Lessons 26–27:** Getting a Handle on New Transformations (E, P)
- Lessons 28–30:** When Can We Reverse a Transformation? (E, E, P)

¹ Lesson Structure Key: **P**-Problem Set Lesson, **M**-Modeling Cycle Lesson, **E**-Exploration Lesson, **S**-Socratic Lesson

The theme of Topic C is to highlight the effectiveness of changing notations and the power provided by certain notations such as matrices. Lessons 18 and 19 exploit the connection to trigonometry, as students see how much of complex arithmetic is simplified (**N-CN.B.4**, **N-CN.B.5**). Students use the connection to trigonometry to solve problems such as *find the three cube roots of -1* . In Lesson 20, complex numbers are regarded as points in the Cartesian plane. If $w = a + ib$, then the modulus is $r = \sqrt{a^2 + b^2}$ and the argument is $\alpha = \arctan(\frac{b}{a})$. Students begin to write analytic formulas for translations, rotations, and dilations in the plane and revisit the ideas of Geometry (**G-CO.A.2**, **G-CO.A.4**, **G-CO.A.5**) in this light. In Lesson 21, students discover a better notation, matrices, and develop the 2×2 matrix notation for planar transformations represented by complex number arithmetic. This work leads to Lessons 22 and 23 as students discover how geometry software and video games efficiently perform rigid motion calculations. Students discover the flexibility of 2×2 matrix notation in Lessons 24 and 25 as they add matrices and multiply by the identity matrix and the zero matrix (**N-VM.C.8**, **N-VM.C.11**). Students understand that multiplying matrix A by the identity matrix results in matrix A and connect the multiplicative identity matrix to the role of 1, the multiplicative identity, in the real number system. This is extended as students see that the identity matrix does not transform the unit square. Students then add matrices and conclude that the zero matrix added to matrix A results in matrix A and is similar to 0 in the real number system. They extend this concept to transformations on the unit square and see that adding the zero matrix has no effect, but multiplying by the zero matrix collapses the unit square to zero. This allows for the study of additional matrix transformations (shears, for example) in Lessons 26 and 27, multiplying matrices, and the meaning of the determinant of a 2×2 matrix (**N-VM.C.10**, **N-VM.C.12**). Lessons 28–30 conclude Topic C and Module 1 as students discover the inverse matrix (matrix A is called an *inverse matrix* to a matrix B if $AB = I$ and $BA = I$) and determine when matrices do not have inverses. Students begin to think and reason abstractly about the geometric effects of the operations of complex numbers (MP.2) as they see the connection to trigonometry and the Cartesian plane.

The study of vectors and matrices is only introduced in Module 1 through a coherent connection to transformations and complex numbers. Further and more formal study of multiplication of matrices will occur in Module 2. **N-M.C.8** will be assessed secondarily, in the context of other standards, but not directly on mid- and end-of-module assessments until Module 2.