Lesson 17: The Geometric Effect of Multiplying by a Reciprocal

Classwork

Opening Exercise

Given . What is and ? Explain how you got your answer.

Exploratory Challenge 1/Exercises 1–9

1. Describe the geometric effect of the transformation .
2. Describe a way to undo the effect of the transformation .
3. Given that for any complex number, how could you describe any clockwise rotation of as an argument of a complex number?
4. Write a complex number in polar form that describes a rotation and dilation that will undo multiplication by , and then convert it to rectangular form.
5. In a previous lesson you learned that to undo multiplication by , you would multiply by the reciprocal . Write the complex number in rectangular form where and are real numbers.
6. How do your answers to Exercises 4 and 5 compare? What does that tell you about the geometric effect of multiplication by the reciprocal of a complex number?
7. Jimmy states the following:

*Multiplication by has the reverse geometric effect of multiplication .*

Do you agree or disagree? Use your work on the previous exercises to support your reasoning.

1. Show that the following statement is true when :

*The reciprocal of a complex number with modulus and argument is with modulus and argument .*

1. Explain using transformations why .

Exploratory Challenge 2/Exercise 10

1. Complete the graphic organizer below to summarize your work with complex numbers so far.

|  |  |  |
| --- | --- | --- |
| **Operation** | **Geometric Transformation** | **Example. Illustrate algebraically and geometrically**  **Let and** |
| Addition |  |  |
| Subtraction |  |  |
| Conjugate of |  |  |
| Multiplication |  |  |
| Reciprocal of |  |  |
| Division |  |  |

**Exercises 11–13**

Let and let . Describe each complex number as a transformation of and then write the number in rectangular form.



Problem Set

1. Describe the geometric effect of multiplying by the reciprocal of each complex number listed below.
2. Let . Show that the geometric transformations you described in Problem 1 really produce the correct complex number by performing the indicated operation and determining the argument and modulus of each number.
3. In Exercise 12 of this lesson you described the complex number as a transformation of for a specific complex number . Show that this transformation always produces a dilation of .
4. Does satisfy the conditions that and which makes it a linear transformation? Justify your answer.
5. Show that describes a reflection of about the line containing the origin and for and   
   .
6. Describe the geometric effect of each transformation function on where , and are complex numbers.
7. Verify your answers to Problem 6 if , , and .