Lesson 17: The Geometric Effect of Multiplying by a Reciprocal

Classwork

Opening Exercise

Given $w=1+i$. What is $arg⁡(w)$ and $\left|w\right|$? Explain how you got your answer.

Exploratory Challenge 1/Exercises 1–9

1. Describe the geometric effect of the transformation $L\left(z\right)=\left(1+i\right)z$.
2. Describe a way to undo the effect of the transformation $L\left(z\right)=\left(1+i\right)z$.
3. Given that $0\leq arg⁡(z)<2π$ for any complex number, how could you describe any clockwise rotation of $θ$ as an argument of a complex number?
4. Write a complex number in polar form that describes a rotation and dilation that will undo multiplication by $(1+i)$, and then convert it to rectangular form.
5. In a previous lesson you learned that to undo multiplication by $1+i$, you would multiply by the reciprocal $\frac{1}{1+i}$. Write the complex number $\frac{1}{1+i}$ in rectangular form $z=a+bi$ where $a$ and $b$ are real numbers.
6. How do your answers to Exercises 4 and 5 compare? What does that tell you about the geometric effect of multiplication by the reciprocal of a complex number?
7. Jimmy states the following:

*Multiplication by* $\frac{1}{a+bi}$ *has the reverse geometric effect of multiplication* $by +bi$*.*

Do you agree or disagree? Use your work on the previous exercises to support your reasoning.

1. Show that the following statement is true when $z=2-2\sqrt{3}i$:

*The reciprocal of a complex number* $z$ *with modulus* $r$ *and argument* $θ$ *is* $\frac{1}{z}$ *with modulus* $\frac{1}{r}$ *and argument* $2π-θ$*.*

1. Explain using transformations why $z∙\frac{1}{z}=1$.

Exploratory Challenge 2/Exercise 10

1. Complete the graphic organizer below to summarize your work with complex numbers so far.

|  |  |  |
| --- | --- | --- |
| **Operation** | **Geometric Transformation** | **Example. Illustrate algebraically and geometrically****Let** $z=3-3i$ **and** $w=-2i$ |
| Addition$$z+w$$ |  |  |
| Subtraction$$z-w$$ |  |  |
| Conjugate of $z$ |  |  |
| Multiplication$$w∙z$$ |  |  |
| Reciprocal of $z$ |  |  |
| Division$$\frac{w}{z}$$ |  |  |

**Exercises 11–13**

Let $z=-1+i$ and let $w=2i$. Describe each complex number as a transformation of $z$ and then write the number in rectangular form.

1. $w\overbar{z}$
2. $\frac{1}{\overbar{z}}$
3. $\overbar{w+z}$

Problem Set

1. Describe the geometric effect of multiplying $z$ by the reciprocal of each complex number listed below.
	1. $w\_{1}=3i$
	2. $w\_{2}=-2$
	3. $w\_{3}=\sqrt{3}+i$
	4. $w\_{4}=1-\sqrt{3}i$
2. Let $z=-2-2\sqrt{3}i$. Show that the geometric transformations you described in Problem 1 really produce the correct complex number by performing the indicated operation and determining the argument and modulus of each number.
	1. $\frac{-2-2\sqrt{3}i}{w\_{1}}$
	2. $\frac{-2-2\sqrt{3}i}{w\_{2}}$
	3. $\frac{-2-2\sqrt{3}i}{w\_{3}}$
	4. $\frac{-2-2\sqrt{3}i}{w\_{4}}$
3. In Exercise 12 of this lesson you described the complex number $\frac{1}{\overbar{z}}$ as a transformation of $z$ for a specific complex number $z$. Show that this transformation always produces a dilation of $z=a+bi$.
4. Does $L\left(z\right)=\frac{1}{z}$ satisfy the conditions that $L\left(z+w\right)=L\left(z\right)+L(w)$ and $L\left(mz\right)=mL(z)$ which makes it a linear transformation? Justify your answer.
5. Show that $L\left(z\right)=w\left(\overbar{\frac{1}{w}z}\right)$ describes a reflection of $z$ about the line containing the origin and $w$ for $z=3i$ and
$w=1+i$.
6. Describe the geometric effect of each transformation function on $z$ where $z$, $w,$ and $a$ are complex numbers.
	1. $L\_{1}\left(z\right)=\frac{z-w}{a}$
	2. $L\_{2}\left(z\right)=\overbar{\left(\frac{z-w}{a}\right)}$
	3. $L\_{3}\left(z\right)=a\overbar{\left(\frac{z-w}{a}\right)}$
	4. $L\_{3}\left(z\right)=a\overbar{\left(\frac{z-w}{a}\right)}+w$
7. Verify your answers to Problem 6 if $z=1-i$, $w=2i$, and $a=-1-i$.
	1. $L\_{1}\left(z\right)=\frac{z-w}{a}$
	2. $L\_{2}\left(z\right)=\overbar{\left(\frac{z-w}{a}\right)}$
	3. $L\_{3}\left(z\right)=a\overbar{\left(\frac{z-w}{a}\right)}$
	4. $L\_{3}\left(z\right)=a\overbar{\left(\frac{z-w}{a}\right)}+w$