# Lesson 16: Representing Reflections with Transformations

## Classwork

### Opening Exercise

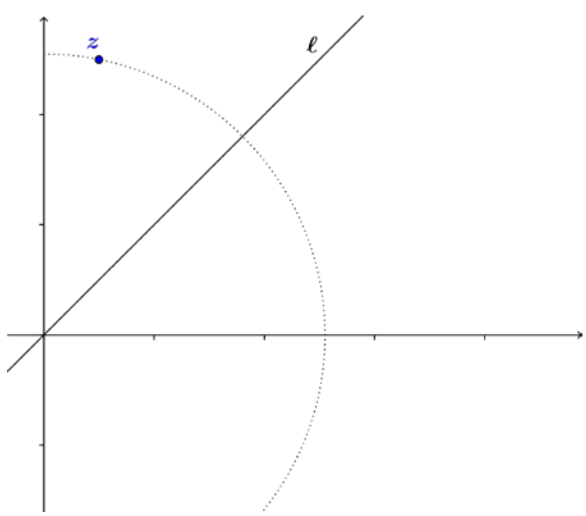
a. Find a transformation $R_{(0,45°)}: \mathbb{C} \to \mathbb{C}$ that rotates a point represented by the complex number $z$ by $45°$ counterclockwise in the coordinate plane, but does not produce a dilation.

b. Find a transformation $R_{(0,-45°)}: \mathbb{C} \to \mathbb{C}$ that rotates a point represented by the complex number $z$ by $45°$ clockwise in the coordinate plane, but does not produce a dilation.

c. Find a transformation $r_{x\text{-axis}}: \mathbb{C} \to \mathbb{C}$ that reflects a point represented by the complex number $z$ across the $x$-axis.

### Discussion

We want to find a transformation $r_\ell: \mathbb{C} \to \mathbb{C}$ that reflects a point representing a complex number $z$ across the diagonal line $\ell$ with equation $y = x$.



## Exercises

1. The number $z$ in the figure used in the discussion above is the complex number $1 + 5i$. Compute $r_\ell(1 + 5i)$ and plot it below.

2. We know from previous courses that the reflection of a point $(x, y)$ across the line with equation $y = x$ is the point $(y, x)$. Does this agree with our result from the previous discussion?

3. We now want to find a formula for the transformation of reflection across the line $\ell$ that makes a $60°$ angle with the positive $x$-axis. Find formulas to represent each component of the transformation, and use them to find one formula that represents the overall transformation.

### Lesson Summary

Let $\ell$ be a line through the origin that contains the terminal ray of a rotation of the $x$-axis by $\theta$. Then reflection across line $\ell$ can be done by the following sequence of transformations:

- Rotation by $-\theta$ about the origin.
- Reflection across the $x$-axis.
- Rotation by $\theta$ about the origin.

## Problem Set

1. Find a formula for the transformation of reflection across the line $\ell$ with equation $y = -x$.

2. Find the formula for the sequence of transformations comprising reflection across the line with equation $y = x$ and then rotation by $180°$ about the origin.

3. Compare your answers to Problems 1 and 2. Explain what you find.

4. Find a formula for the transformation of reflection across the line $\ell$ that makes a $-30°$ angle with the positive $x$-axis.

5. Max observed that when reflecting a complex number, $z = a + bi$ about the line $y = x$, that $a$ and $b$ are reversed, which is similar to how we learned to find an inverse function. Will Max's observation also be true when the line $y = -x$ is used, where $a = -b$ and $= -a$ ? Give an example to show his assumption is either correct or incorrect.

6. For reflecting a complex number, $z = a + bi$ about the line $y = 2x$, will Max's idea work if he makes $b = 2a$ and $a = \frac{b}{2}$? Use $z = 1 + 4i$ as an example to show whether or not it works.

7. What would the formula look like if you want to reflect a complex number about the line $y = mx$, where $m > 0$?