Lesson 16: Representing Reflections with Transformations

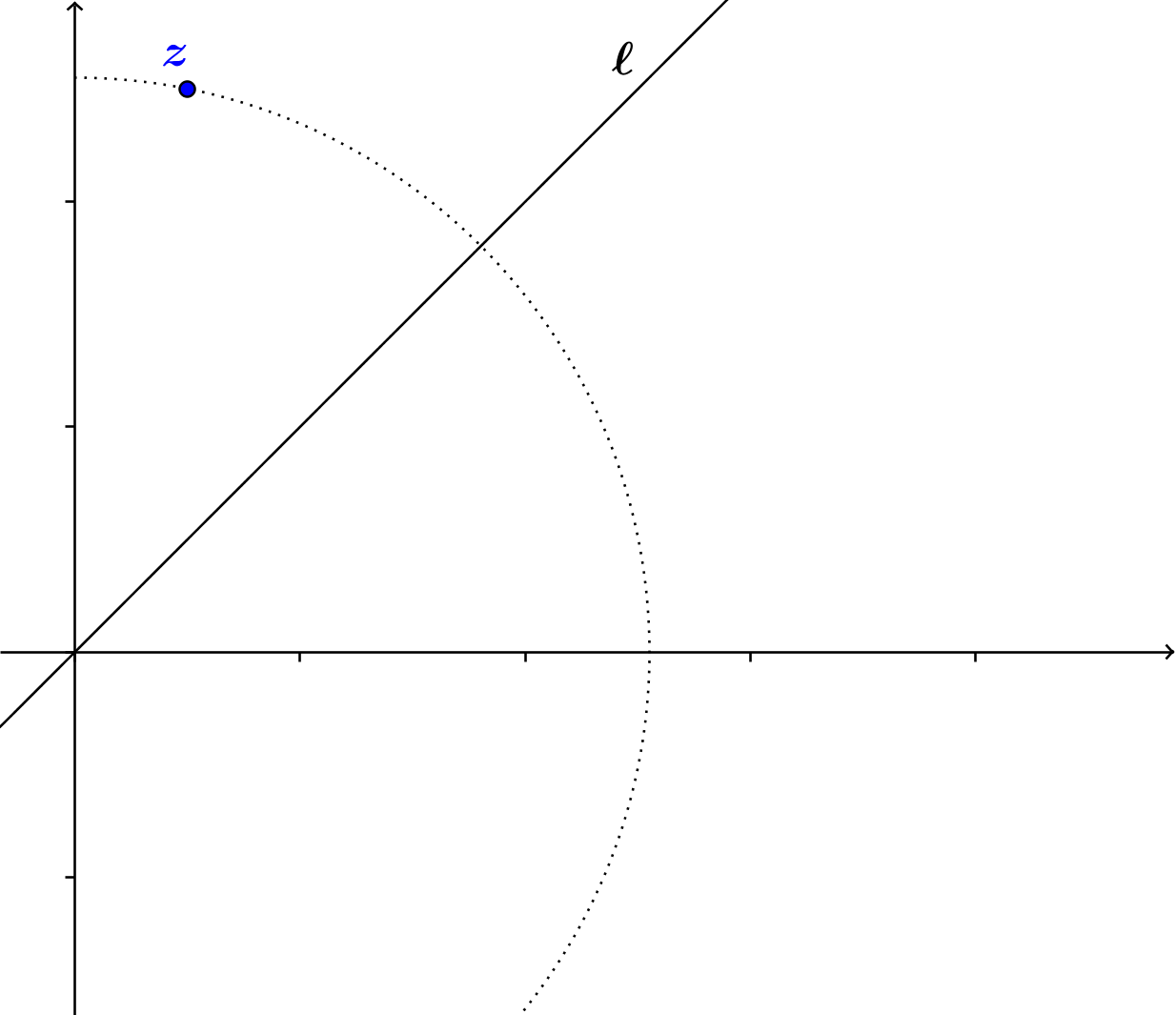
Classwork

Opening Exercise

* 1. Find a transformation that rotates a point represented by the complex number by counterclockwise in the coordinate plane, but does not produce a dilation.
  2. Find a transformation that rotates a point represented by the complex number by clockwise in the coordinate plane, but does not produce a dilation.
  3. Find a transformation that reflects a point represented by the complex number across the   
     -axis.

Discussion

We want to find a transformation that reflects a point representing a complex number across the diagonal line with equation.



Exercises

1. The number in the figure used in the discussion above is the complex number . Compute and plot it below.
2. We know from previous courses that the reflection of a point across the line with equationis the point . Does this agree with our result from the previous discussion?
3. We now want to find a formula for the transformation of reflection across the line that makes a angle with the positive -axis. Find formulas to represent each component of the transformation, and use them to find one formula that represents the overall transformation.

Lesson Summary

Let be a line through the origin that contains the terminal ray of a rotation of the -axis by . Then reflection across line can be done by the following sequence of transformations:

* Rotation by about the origin.
* Reflection across the -axis.
* Rotation by about the origin.

Problem Set

1. Find a formula for the transformation of reflection across the line with equation .
2. Find the formula for the sequence of transformations comprising reflection across the line with equation and then rotation by about the origin.
3. Compare your answers to Problems 1 and 2. Explain what you find.
4. Find a formula for the transformation of reflection across the line that makes a angle with the positive -axis.
5. Max observed that when reflecting a complex number, about the line , that and are reversed, which is similar to how we learned to find an inverse function. Will Max’s observation also be true when the line   
    is used, where and ? Give an example to show his assumption is either correct or incorrect.
6. For reflecting a complex number, about the line , will Max’s idea work if he makes and ? Use as an example to show whether or not it works.
7. What would the formula look like if you want to reflect a complex number about the line , where ?