

Lesson 12: Distance and Complex Numbers

Classwork

Opening Exercise

- Let $A = 2 + 3i$ and $B = -4 - 8i$. Find a complex number C so that B is the midpoint of A and C .
- Given two complex numbers A and B , find a formula for a complex number C in terms of A and B so that B is the midpoint of A and C .
- Verify that your formula is correct by using the result of part (a).

Exercise

Let $z = -100 + 100i$ and $w = 1000 - 1000i$.

- Find a point one quarter of the way along the line segment connecting z and w closer to z than to w .
- Write this point in the form $\alpha z + \beta w$ for some real numbers α and β . Verify that this does in fact represent the point found in part (a).
- Describe the location of the point $\frac{2}{5}z + \frac{3}{5}w$ on this line segment.

Exploratory Challenge 1

- a. Draw three points A , B , and C in the plane.
- b. Start at any position P_0 and leapfrog over A to a new position P_1 so that A is the midpoint of $\overline{P_0P_1}$.
- c. From P_1 , leapfrog over B to a new position P_2 so that B is the midpoint $\overline{P_1P_2}$.
- d. From P_2 , leapfrog over C to a new position P_3 so that C is the midpoint $\overline{P_2P_3}$.
- e. Continue alternately leapfrogging over A , then B , then C .
- f. What eventually happens?

- g. Using the formula from Opening Exercise part (b), show why this happens.

Exploratory Challenge 2

- a. Plot a single point A in the plane.
- b. What happens when you repeatedly jump over A ?
- c. Using the formula from Opening Exercise part (b), show why this happens.
- d. Make a conjecture about what will happen if you leapfrog over two points, A and B , in the coordinate plane.
- e. Test your conjecture by using the formula from Opening Exercise part (b).
- f. Was your conjecture correct? If not, what is your new conjecture about what happens when you leapfrog over two points, A and B , in the coordinate plane?
- g. Test your conjecture by actually conducting the experiment.

Problem Set

- Find the distance between the following points.
 - Point $A(2, 3)$ and point $B(6, 6)$
 - $A = 2 + 3i$ and $B = 6 + 6i$
 - $A = -1 + 5i$ and $B = 5 + 11i$
 - $A = 1 - 2i$ and $B = -2 + 3i$
 - $A = \frac{1}{2} - \frac{1}{2}i$ and $B = -\frac{2}{3} + \frac{1}{3}i$
- Given three points A, B, C , where C is the midpoint of A and B .
 - If $A = -5 + 2i$ and $C = 3 + 4i$, find B .
 - If $B = 1 + 11i$ and $C = -5 + 3i$, find A .
- Point C is the midpoint between $A = 4 + 3i$ and $B = -6 - 5i$. Find the distance between points C and D for each point D provided below.
 - $2D = -6 + 8i$
 - $D = -\bar{B}$
- The distance between points $A = 1 + 1i$ and $B = a + bi$ is 5. Find the point B for each value provided below.
 - $a = 4$
 - $b = 6$
- Draw five points in the plane A, B, C, D, E . Start at any position, P_0 , and leapfrog over A to a new position, P_1 (so, A is the midpoint of $\overline{P_0P_1}$). Then leapfrog over B , then C , then D , then E , then A , then B , then C , then D , then E , then A again, and so on. How many jumps will it take to get back to the start position, P_0 ?
- For the leapfrog puzzle problems in both Exploratory Challenge 1 and Problem 5, we are given an odd number of points to leapfrog over. What if we leapfrog over an even number of points? Let $A = 2, B = 2 + i$, and $P_0 = i$. Will P_n ever return to the starting position, P_0 ? Explain how you know.