Lesson 12: Distance and Complex Numbers

Classwork

Opening Exercise

* 1. Let and. Find a complex number so that is the midpoint of and .
  2. Given two complex numbers and , find a formula for a complex number in terms of and so that is the midpoint of and .
  3. Verify that your formula is correct by using the result of part (a).

Exercise

Let and .

* 1. Find a point one quarter of the way along the line segment connecting and closer to than to .
  2. Write this point in the form for some real numbers and . Verify that this does in fact represent the point found in part (a).
  3. Describe the location of the point on this line segment.

Exploratory Challenge 1

* 1. Draw three points , , and in the plane.
  2. Start at any position and leapfrog over to a new position so that is the midpoint of .
  3. From , leapfrog over to a new position so that is the midpoint .
  4. From , leapfrog over to a new position so that is the midpoint .
  5. Continue alternately leapfrogging over , then, then
  6. What eventually happens?
  7. Using the formula from Opening Exercise part (b), show why this happens.

Exploratory Challenge 2

* 1. Plot a single point in the plane.
  2. What happens when you repeatedly jump over ?
  3. Using the formula from Opening Exercise part (b), show why this happens.
  4. Make a conjecture about what will happen if you leapfrog over two points, and , in the coordinate plane.
  5. Test your conjecture by using the formula from Opening Exercise part (b).
  6. Was your conjecture correct? If not, what is your new conjecture about what happens when you leapfrog over two points, and , in the coordinate plane?
  7. Test your conjecture by actually conducting the experiment.

Problem Set

1. Find the distance between the following points.
   1. Point and point
   2. and
   3. and
   4. and
   5. and
2. Given three points , , , where is the midpoint of and .
   1. If and , find .
   2. If and , find .
3. Point is the midpoint between and . Find the distance between points and for each point provided below.
4. The distance between points and is . Find the point for each value provided below.
5. Draw five points in the plane ,,,,. Start at any position, , and leapfrog over to a new position, (so, is the midpoint of ). Then leapfrog over , then , then , then , then , then , then , then , then , then again, and so on. How many jumps will it take to get back to the start position, ?
6. For the leapfrog puzzle problems in both Exploratory Challenge 1 and Problem 5, we are given an odd number of points to leapfrog over. What if we leapfrog over an even number of points? Let , , and . Will ever return to the starting position, ? Explain how you know.