Lesson 12: Distance and Complex Numbers

Classwork

Opening Exercise

* 1. Let $A=2+3i$ and$ B=-4-8i$. Find a complex number $C$ so that $B$ is the midpoint of $A$ and $C$.
	2. Given two complex numbers $A$ and $B$, find a formula for a complex number $C$ in terms of $A$ and $B$ so that $B $is the midpoint of $A$ and $C$.
	3. Verify that your formula is correct by using the result of part (a).

Exercise

Let $z=-100+100i$ and $w=1000-1000i$.

* 1. Find a point one quarter of the way along the line segment connecting $z $and $w$ closer to $z $than to $w$.
	2. Write this point in the form $αz+βw$ for some real numbers $α $and $β$. Verify that this does in fact represent the point found in part (a).
	3. Describe the location of the point $\frac{2}{5}z+\frac{3}{5}w$ on this line segment.

Exploratory Challenge 1

* 1. Draw three points $A$, $B$, and $C$ in the plane.
	2. Start at any position $P\_{0}$ and leapfrog over $A$ to a new position $P\_{1}$ so that $A$ is the midpoint of $\overbar{P\_{0}P\_{1}}$.
	3. From $P\_{1}$, leapfrog over $B$ to a new position $P\_{2}$ so that $B$ is the midpoint $\overbar{P\_{1}P\_{2}}$.
	4. From $P\_{2}$, leapfrog over $C$ to a new position $P\_{3}$ so that $C$ is the midpoint $\overbar{P\_{2}P\_{3}}$.
	5. Continue alternately leapfrogging over $A$, then$ B$, then $C.$
	6. What eventually happens?
	7. Using the formula from Opening Exercise part (b), show why this happens.

Exploratory Challenge 2

* 1. Plot a single point $A $in the plane.
	2. What happens when you repeatedly jump over $A$?
	3. Using the formula from Opening Exercise part (b), show why this happens.
	4. Make a conjecture about what will happen if you leapfrog over two points, $A$ and $B$, in the coordinate plane.
	5. Test your conjecture by using the formula from Opening Exercise part (b).
	6. Was your conjecture correct? If not, what is your new conjecture about what happens when you leapfrog over two points, $A$ and $B$, in the coordinate plane?
	7. Test your conjecture by actually conducting the experiment.

Problem Set

1. Find the distance between the following points.
	1. Point $A(2, 3)$ and point $B(6, 6) $
	2. $A=2+3i$ and $B=6+6i$
	3. $A=-1+5i $and $B=5+11i$
	4. $A=1-2i $and $B=-2+3i$
	5. $A=\frac{ 1 }{2}-\frac{ 1 }{2}i $and $B=-\frac{ 2 }{ 3 }+\frac{1}{ 3 }i$
2. Given three points $A$, $B$, $C$, where $C$ is the midpoint of $A$ and $B$.
	1. If $A=-5+2i$ and $C=3+4i$, find $B$.
	2. If$B=1+11i$ and $C=-5+3i$, find $A$.
3. Point $C$ is the midpoint between $A=4+3i$ and $B=-6-5i$. Find the distance between points $C$ and $D$ for each point $D$ provided below.
	1. $2D=-6+8i$
	2. $D=-\overbar{B}$
4. The distance between points $A=1+1i$ and $B=a+bi$ is $5$. Find the point $B$ for each value provided below.
	1. $a=4$
	2. $b=6$
5. Draw five points in the plane $A$,$ B$,$ C$,$ D$,$ E$. Start at any position, $P\_{0}$, and leapfrog over $A$ to a new position, $P\_{1}$ (so, $A$ is the midpoint of $\overbar{P\_{0}P\_{1}}$). Then leapfrog over $B$, then $C$, then $D$, then $E$, then $A$, then $B$, then $C$, then $D$, then $E$, then $A$ again, and so on. How many jumps will it take to get back to the start position, $P\_{0}$?
6. For the leapfrog puzzle problems in both Exploratory Challenge 1 and Problem 5, we are given an odd number of points to leapfrog over. What if we leapfrog over an even number of points? Let $A=2$, $B=2+i$, and $P\_{0}=i$. Will $P\_{n}$ ever return to the starting position, $P\_{0}$? Explain how you know.