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Lesson 11: Distance and Complex Numbers

Student Outcomes

* Students calculate distances between complex numbers as the modulus of the difference.
* Students calculate the midpoint of a segment as the average of the numbers at its endpoints.

Lesson Notes

In Topic A, students saw that complex numbers have geometric interpretations associated with them since points in the complex plane seem analogous to points in the coordinate plane. In Lesson 6, students considered complex numbers as vectors and learned to add them by the tip-to-tail method. In Lessons 8 and 9, students explored the idea that every complex operation must have some geometric interpretation, eventually coming to the realization that complex addition and subtraction has the geometric effect of performing a translation to points in the complex plane. The geometric interpretation of complex multiplication was left unresolved as students realized it was not readily obvious. Later in the module, students will continue to explore the question, “What is the geometric action of multiplication by a complex number on all the points in the complex plane?” To understand this, students will first explore the connection between geometry and complex numbers. The coordinate geometry we studied in Geometry was about points in the coordinate plane, whereas now we are thinking about complex numbers in the complex plane.

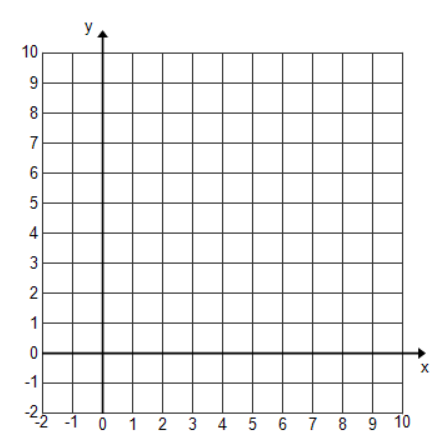
Classwork

Opening Exercise (5 minutes)

Give students time to work independently on the Opening Exercise before discussing as a class.

Opening Exercise

* 1. Plot the complex number on the complex plane. Plot the ordered pair on the coordinate plane.



* 1. In what way are complex numbers “points”?

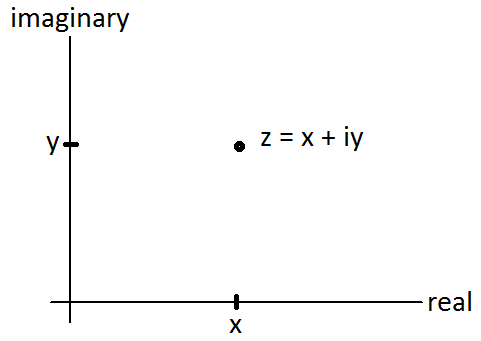
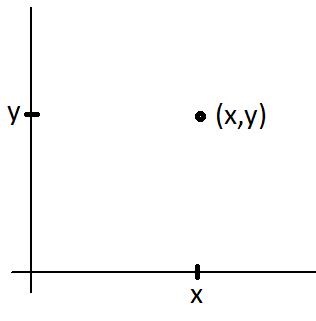
When a complex number is plotted on a complex plane, it looks just like the corresponding ordered pair plotted on a coordinate plane. For example, when is plotted on the complex plane, it looks exactly the same as when the ordered pair is plotted on a coordinate plane. We can interchangeably think of a complex number in the complex plane as a point in the coordinate plane, and vice versa.

* 1. What point on the coordinate plane corresponds to the complex number?
  2. What complex number corresponds to the point located at coordinate?

or

Discussion (7 minutes)

Draw the following on the board.



* When we say that complex numbers are points in the complex plane, what do we really mean?
  + *When a complex number is plotted on the complex plane, it looks exactly the same as when the ordered pair is plotted on a coordinate plane.*
* What does this mean in terms of connecting the ideas we learned in Geometry to complex numbers?
  + *Since we can interchangeably think of a complex number in the complex plane as a point in the coordinate plane, and vice versa, all the work we did in Geometry can be translated into the language of complex numbers, and vice versa. Therefore, any work we do with complex numbers should translate back to results from Geometry.*
* In Geometry, it did not make sense to add two points together. If and are points, what would the geometric meaning of be?

*Scaffolding:*

If needed, provide students with an example using specific numbers rather than general parameters.

* If and   
  , then what is ?
  + *It does not seem to have any meaning.*
* It does make sense to add two complex numbers together. If and , then what is ?
  + +
* What is the geometric effect of transforming with the function for constant complex number ?

**MP.7**

* + *Applying the transformation to has the geometric effect of performing a translation. So, adding to will shift point right units and   
    up .*
* If we view points in the plane as complex numbers, then we can add points in geometry.

Exercise 1 (3 minutes)

Have students work on Exercise 1 independently and then share results with a partner. If students do not recall how to find the midpoint, have them draw the line segment and locate the midpoint from the graph rather than providing them with the midpoint formula.

**Exercises**

1. **The endpoints of are and . What is the midpoint of ?**

**The midpoint of is .**

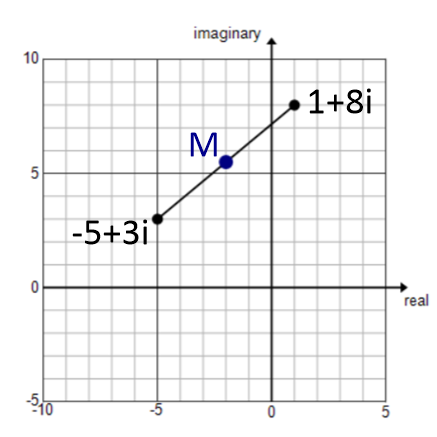
* How do you find the midpoint of ?
  + *You find the average of the -coordinates and the -coordinates to find the halfway point.*
* In Geometry, we learned that for two points and the midpoint of is .
* Now, view these points as complex numbers: and

Exercise 2 (7 minutes)

Allow students time to work on part (a) independently and then share results with a partner. Have them work in partners on part (b) before discussing as a class.

*Scaffolding:*

* Provide visual learners with a graph on the complex plane.



* If students need additional practice, use this example before completing part (b) of Exercise 2:

What is the midpoint of and ?

* 1. **What is the midpoint of and ?**

**MP.2**

* 1. **Using**  and , s**how that in general** the midpoint of points and is , the arithmetic average of the two numbers.

Exercise 3 (5 minutes)

As with Exercise 1, have students work on Exercise 3 independently and then share results with a partner. If students do not recall how to find the length of a line segment, have them draw the line segment and instruct them to think of it as the hypotenuse of a right triangle rather than providing them with the distance formula.

1. **The endpoints of are and . What is the length of ?**

**The length of  *is .***

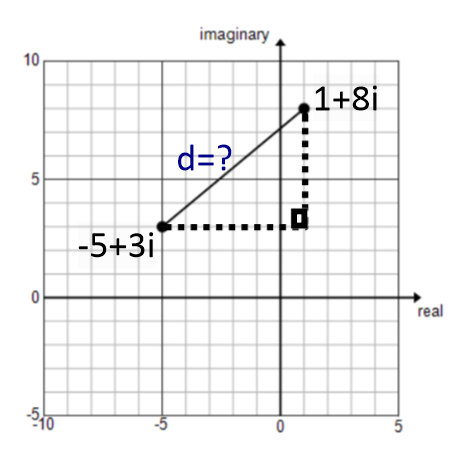
* How do you find the length of ?
  + *You use the Pythagorean theorem which can be written as for two points and*
* As we did previously, view these points as complex numbers: and

Exercise 4 (7 minutes)

As with Exercise 2, allow students time to work on part (a) independently and then share results with a partner. Have them work in partners on part (b) before discussing as a class.

*Scaffolding:*

* Provide visual learners with a graph on the complex plane.



* If students need additional practice, use this example before completing part (b) of Exercise 4:

What is the distance between   
and?

* 1. **What is the distance between and ?**

**MP.2**

* 1. Show that, in general, the distance between and   
      is the modulus of .

**distance between and .**

Exercise 5 (3 minutes)

Allow students time to work either independently or with a partner. Circulate the room to ensure that students understand the concepts.

1. Suppose and .

*Scaffolding:*

Have struggling students create a graphic organizer comparing a coordinate plane and a complex plane.

|  |  |  |
| --- | --- | --- |
|  | Coordinate plane | Complex plane |
|  | and | and |
| Graph |  |  |
| Midpoint |  |  |
| Distance |  |  |

* 1. Find the midpoint of and .
  2. Verify that .

Closing (3 minutes)

Have students discuss each question with a partner. Then elicit class responses.

* In what way can complex numbers be thought of as points?
  + *When a complex number is plotted on a complex plane, it looks just like the corresponding ordered pair plotted on a coordinate plane.*
* Why is it helpful to interchange between complex numbers and points on a plane?
  + *Unlike points on a plane, we can add and subtract complex numbers. Thus, we can use operations on complex numbers to find geometric measurements such as midpoint and distance.*

Lesson Summary

* **Complex numbers can be thought of as points in a plane, and points in a plane can be thought of as complex numbers.**
* For two complex numbers  **and , the midpoint of points and is .**
* **The distance between two complex numbers and is equal to**

Exit Ticket (5 minutes)

Name Date

Lesson 11: Distance and Complex Numbers

Exit Ticket

1. Kishore said that he can add two points in the coordinate plane like adding complex numbers in the complex plane. For example, for point and point , he will get . Is he correct? Explain your reasoning.
2. Consider two complex numbers and .
   1. Find the midpoint of and .
   2. Find the distance between and .

Exit Ticket Sample Solutions

1. Kishore said that he can add two points in the coordinate plane like adding complex numbers in the complex plane. For example, for point and point , he will get . Is he correct? Explain your reasoning.

No. Kishore is not correct because we cannot add two points in the rectangular plane. However, we can add two complex numbers in the complex plane, which has the geometric effect of performing a translation to points in complex numbers.

1. Consider two complex numbers and .
   1. Find the midpoint of and .
   2. Find the distance between and **.**

Problem Set Sample Solutions

1. Find the midpoint between the two given points in the rectangular coordinate plane.
   1. and
   2. and
   3. and
   4. and
   5. and
   6. and
2. Let , , and suppose that is the midpoint of and , and that is the midpoint of and .
   1. Find points and .
   2. Find the distance between and .
   3. Find the distance between and .
   4. Find the distance between and .
   5. Find the distance between and .
   6. Find a point one quarter of the way along the line segment connecting segment and , closer to than   
      to .

The point is .

* 1. Terrence thinks the distance from to is the same as the distance from to . Is he correct? Explain why or why not.

The distance from to is , and the distance from to is . The distances are not the same.

* 1. Using your answer from part (g), if is the midpoint of and , can you find the distance from to ? Explain.

The distance from to is , and the distance from to should be half of this value, .

* 1. Without doing any more work, can you find point ? Explain.

is , which is units to the right of in the real direction and unit up in the imaginary direction. From , you should move the same amount to get to , so would be .