Lesson 11: Distance and Complex Numbers

Classwork

Opening Exercise

* 1. Plot the complex number $z=2+3i$ on the complex plane. Plot the ordered pair $(2, 3)$ on the coordinate plane.



* 1. In what way are complex numbers “points”?
	2. What point on the coordinate plane corresponds to the complex number$ -1+8i$?
	3. What complex number corresponds to the point located at coordinate$ (0,-9)$?

**Exercises**

1. The endpoints of a $\overbar{AB}$ are $A(1, 8)$ and $B(-5, 3)$. What is the midpoint of $\overbar{AB}$?
	1. What is the midpoint of $A=1+8i$ and $B=-5+3i$?
	2. Using $A=x\_{1}+y\_{1}i$ and $B=x\_{2}+y\_{2}i,$ show that in general the midpoint of points $A$ and $B$ is $\frac{A+B}{2}$, the arithmetic average of the two numbers.
2. The endpoints of $\overbar{AB}$ are $A(1, 8)$ and $B(-5, 3)$. What is the length of $\overbar{AB}$?
	1. What is the distance between $A=1+8i$ and $B=-5+3i$?
	2. Show that, in general, the distance between $A=x\_{1}+y\_{1}i$ and $B=x\_{2}+y\_{2}i$ is the modulus of $A-B.$
3. Suppose $z=2+7i$ and $w=-3+i$.
	1. Find the midpoint $m$ of $z$ and $w$.
	2. Verify that $\left|z-m\right|=\left|w-m\right|$.

Lesson Summary

* Complex numbers can be thought of as points in a plane, and points in a plane can be thought of as complex numbers.
* For two complex numbers $A=x\_{1}+y\_{1}i$ and $B=x\_{2}+y\_{2}i$, the midpoint of points $A$ and $B$ is $\frac{A+B}{2}$.
* The distance between two complex numbers $A=x\_{1}+y\_{1}i$ and $B=x\_{2}+y\_{2}i $is equal to $\left|A-B\right|.$

Problem Set

1. Find the midpoint between the two given points in the rectangular coordinate plane.
	1. $2+4i$ and $4+8i$
	2. $-3+7i$ and $5-i$
	3. $-4+3i$ and $9-4i$
	4. $4+i$ and $-12-7i$
	5. $-8-3i$ and $3-4i$
	6. $\frac{2}{3}-\frac{5}{2}i$ and $-0.2+0.4i$
2. Let $A=2+4i$, $B=14+8i$, and suppose that $C$ is the midpoint of $A$ and $B$, and that $D$ is the midpoint of $A$ and $C$.
	1. Find points $C$ and $D$.
	2. Find the distance between $A$ and $B$.
	3. Find the distance between $A$ and $C$.
	4. Find the distance between $C$ and $D$.
	5. Find the distance between $D$ and $B$.
	6. Find a point one quarter of the way along the line segment connecting segment $A$ and $B$, closer to $A$ than
	to $B$.
	7. Terrence thinks the distance from $B$ to $C$ is the same as the distance from $A$ to $B$. Is he correct? Explain why or why not.
	8. Using your answer from part (g), if $E$ is the midpoint of $C$ and $B$, can you find the distance from$ E$ to $C$? Explain.
	9. Without doing any more work, can you find point $E$? Explain.