



Topic B:

Complex Number Operations and Transformations

N-CN.A.3, N-CN.B.4, N-CN.B.5, N-CN.B.6

Focus Standards:	N-CN.A.3	(+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
	N-CN.B.4	(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
	N-CN.B.5	(+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has a modulus of 2 and an argument of 120° .
	N-CN.B.6	(+) Calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the number at its endpoints.
Instructional Days:	9	
Lessons 9–10:	The Geometric Effect of Some Complex Arithmetic (P, P) ¹	
Lessons 11–12:	Distance and Complex Numbers (P, E)	
Lesson 13:	Trigonometry and Complex Numbers (P)	
Lesson 14:	Discovering the Geometric Effect of Complex Multiplication (E)	
Lesson 15:	Justifying the Geometric Effect of Complex Multiplication (S)	
Lesson 16:	Representing Reflections with Transformations (P)	
Lesson 17:	The Geometric Effect of Multiplying by a Reciprocal (E)	

In Topic B, students develop an understanding of the geometric effect of operations on complex numbers. In Lesson 9, students explore what happens to a point in the complex plane when complex numbers are added and subtracted leading to Lesson 10's study of the effect of multiplication (**N-CN.B.5**). Students revisit the

¹ Lesson Structure Key: **P**-Problem Set Lesson, **M**-Modeling Cycle Lesson, **E**-Exploration Lesson, **S**-Socratic Lesson

idea of linearity in Lesson 10, determining whether given complex functions are linear transformations. Students discover that some complex functions are linear transformations and others are not. Students understand that when complex numbers are considered points in the Cartesian plane, complex number multiplication has the geometric effect of a rotation followed by a dilation in the complex plane. In Lesson 11, students use the distance and midpoint formulas they studied in Geometry to find the distance of a complex point from the origin and the midpoint of two complex numbers (**N-CN.B.6**). Lesson 12 extends this concept as students play the Leap Frog game, repeatedly finding the midpoints of pairs of complex numbers. They discover that when starting with three fixed midpoints, a series of moves jumping across midpoints leads back to the starting point. Students then explore what happens when they start with only one or two fixed midpoints and find that a series of jumps never returns to the starting point. Students verify these results using the midpoint formula derived earlier in the lesson. Lesson 13 introduces the modulus and argument and polar coordinates as students study the geometric effect of complex number multiplication leading to writing the complex number in polar form (**N-CN.B.4**). If a complex number z has argument θ and modulus r , then it can be written in polar form, $z = r(\cos \theta + i \sin \theta)$. Students explain why complex numbers can be written in either rectangular or polar form and why the two forms are equivalent. Lessons 14–16 continue the study of multiplication by complex numbers, leading students to the understanding that the geometric effect of multiplying by a complex number, w , is a rotation of the argument of w followed by a dilation with scale factor the modulus of w . Lesson 17 concludes this topic as students discover that the multiplicative inverse of a complex number (i.e., its reciprocal) provides the inverse geometric operation which leads to the complex conjugate and division of complex numbers (**N-CN.A.3**). Mathematical practices highlighted in the topic include MP.2 as students reason abstractly and quantitatively about complex numbers and the geometric effects of operations involving complex numbers and use previous results to predict rotations and dilations produced.