Lesson 8: Complex Number Division

Classwork

Opening Exercises

Use the general formula to find the multiplicative inverse of each complex number.

a. $2 + 3i$

b. $-7 - 4i$

c. $-4 + 5i$

Exercises 1–4

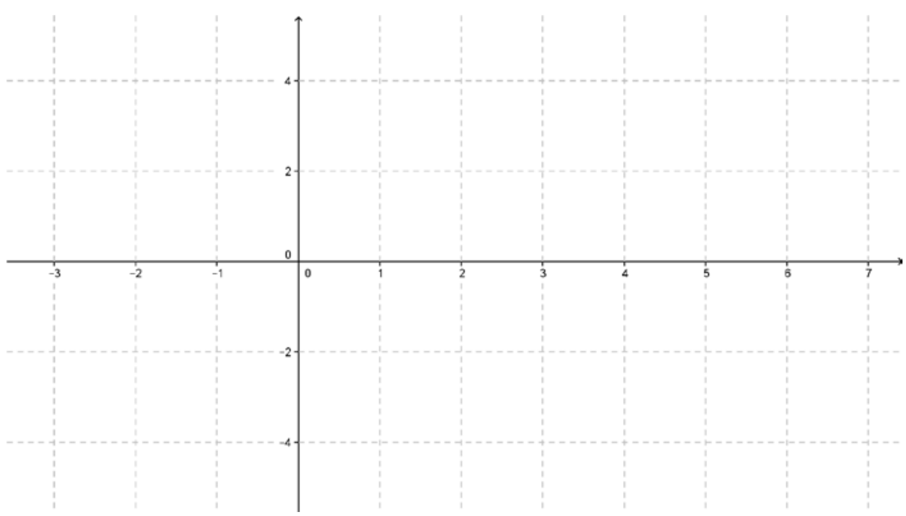
Find the conjugate, and plot the complex number and its conjugate in the complex plane. Label the conjugate with a prime symbol.

1. A: $3 + 4i$

2. B: $-2 - i$

3. C: 7

4. D: $4i$



Exercises 5–8

Find the modulus.

5. $3 + 4i$

6. $-2 - i$

7. 7

8. $4i$

Exercises 9–11

Given $z = a + bi$.

9. Show that for all complex numbers z, $|iz| = |z|$.

10. Show that for all complex numbers z, $z \cdot \bar{z} = |z|^2$.

11. Explain the following: Every nonzero complex number z has a multiplicative inverse. It is given by $\frac{1}{z} = \frac{\bar{z}}{|z|}$.

Example 1

$$\frac{2-6i}{2+5i}$$

Exercises 12–13

Divide.

12. $\frac{3+2i}{-2-7i}$

13. $\frac{3}{3-i}$

Problem Set

1. Let $z = 4 - 3i$ and $w = 2 - i$. Show that
   a. $|z| = |\overline{z}|$
   b. $\left|\frac{1}{z}\right| = \frac{1}{|\overline{z}|}$
   c. If $|z| = 0$, must it be that $z = 0$?
   d. Give a specific example to show that $|z + w|$ usually does not equal $|z| + |w|$.

2. Divide.
   a. $\frac{1 - 2i}{2i}$
   b. $\frac{5 - 2i}{5 + 2i}$
   c. $\frac{\sqrt{3} - 2i}{-2 - \sqrt{3}i}$

3. Prove that $|zw| = |z| \cdot |w|$ for complex numbers z and w.

4. Given $z = 3 + i$, $w = 1 + 3$.
   a. Find $z + w$, and graph z, w, and $z + w$ on the same complex plane. Explain what you discover if you draw line segments from the origin to those points z,w, and $z + w$. Then draw line segments to connect w to $z + w$, and $z + w$ to z.
   b. Find $-w$, and graph z, w, and $z - w$ on the same complex plane. Explain what you discover if you draw line segments from the origin to those points z, w, and $z - w$. Then draw line segments to connect w to $z - w$, and $z - w$ to z.

5. Explain why $|z + w| \leq |z| + |w|$ and $|z - w| \leq |z| + |w|$ geometrically. (Hint: Triangle inequality theorem)