Lesson 8: Complex Number Division

Classwork

Opening Exercises

Use the general formula to find the multiplicative inverse of each complex number.

* 1. $2+3i$
	2. $-7-4i$
	3. $-4+5i$

Exercises 1–4

Find the conjugate, and plot the complex number and its conjugate in the complex plane. Label the conjugate with a prime symbol.

1. $A$: $3+4i$
2. $B$: $-2-i$
3. $C$: $7$
4. $D$: $4i$

Exercises 5–8

Find the modulus.

1. $3+4i$
2. $-2-i$
3. $7$
4. $4i$

Exercises 9–11

Given $z=a+bi$.

1. Show that for all complex numbers $z$, $\left|iz\right|=\left|z\right|$.
2. Show that for all complex numbers $z$, $z∙\overbar{z}=\left|z\right|^{2}$.
3. Explain the following: Every nonzero complex number $z$ has a multiplicative inverse. It is given by $\frac{1}{z}=\frac{\overbar{z}}{\left|z\right|}$.

**Example 1**

$$\frac{2-6i}{2+5i}$$

Exercises 12–13

Divide.

1. $\frac{3 + 2i}{-2 - 7i}$
2. $\frac{3}{3 - i}$

Problem Set

1. Let $z=4-3i$ and$w=2-i$. Show that
	1. $\left|z\right|=\left|\overbar{z}\right|$
	2. $\left|\frac{1}{z}\right|=\frac{1}{\left|\overbar{z}\right|} $
	3. If$\left|z\right|=0$, must it be that $z=0$?
	4. Give a specific example to show that $\left|z+w\right|$ usually does not equal $\left|z\right|+\left|w\right|$.
2. Divide.
	1. $\frac{1 - 2i}{2i}$
	2. $\frac{5 - 2i}{5 + 2i}$
	3. $\frac{\sqrt{3} - 2i}{-2 - \sqrt{3}i}$
3. Prove that $\left|zw\right|=\left|z\right|∙\left|w\right|$ for complex numbers $z$ and $w$.
4. Given $z=3+i$, $w=1+3$.
	1. Find $z+w$, and graph $z$, $w$, and $z+w$ on the same complex plane. Explain what you discover if you draw line segments from the origin to those points $z$,$w$, and $z+w$. Then draw line segments to connect $w$ to $z+w$, and $z+w$ to $z$.
	2. Find $-w$, and graph $z$, $w$, and $z-w$ on the same complex plane. Explain what you discover if you draw line segments from the origin to those points $z$,$ w$, and $z-w$. Then draw line segments to connect $w$ to $z-w$, and $z-w$ to $z$.
5. Explain why $\left|z+w\right|\leq \left|z\right|+\left|w\right|$ and $\left|z-w\right|\leq \left|z\right|+\left|w\right|$ geometrically. (Hint: Triangle inequality theorem)