Lesson 6: Complex Numbers as Vectors

Classwork

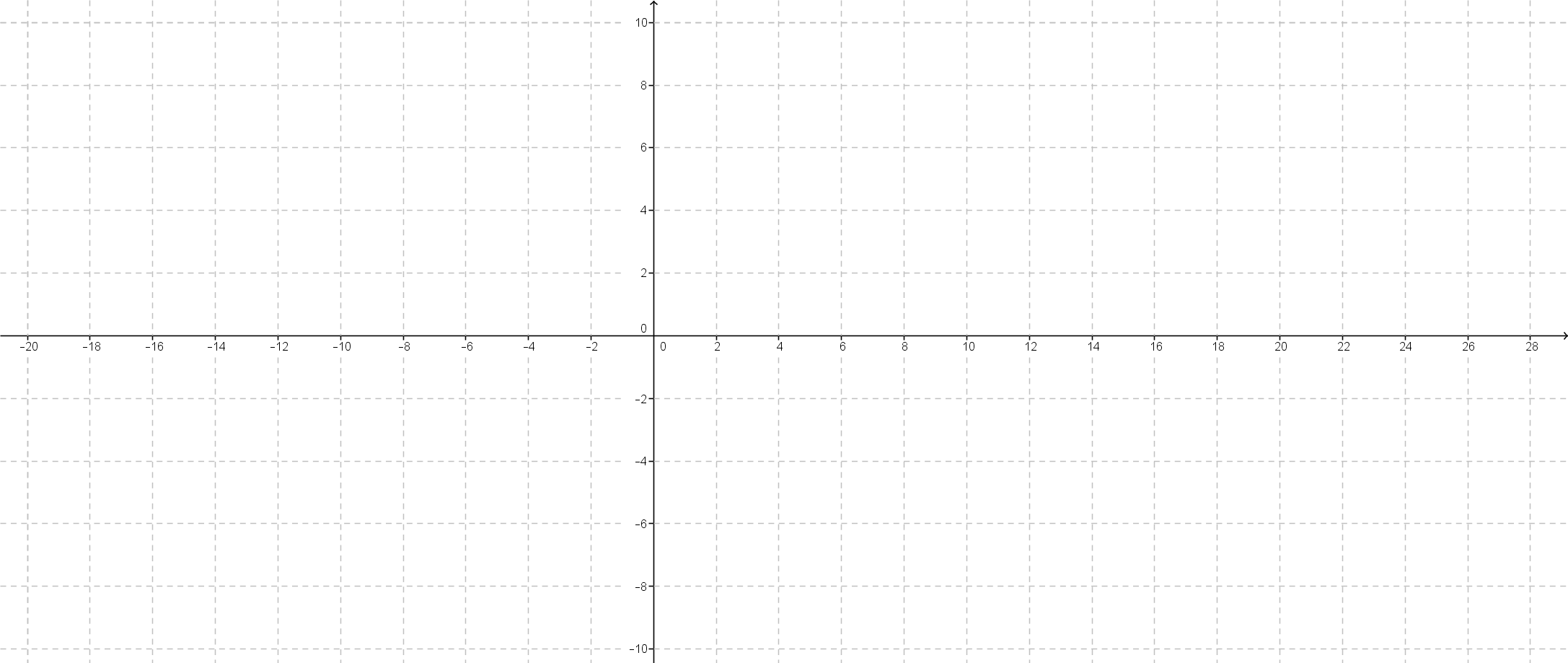
Opening Exercises

Perform the indicated arithmetic operations for complex numbers and .

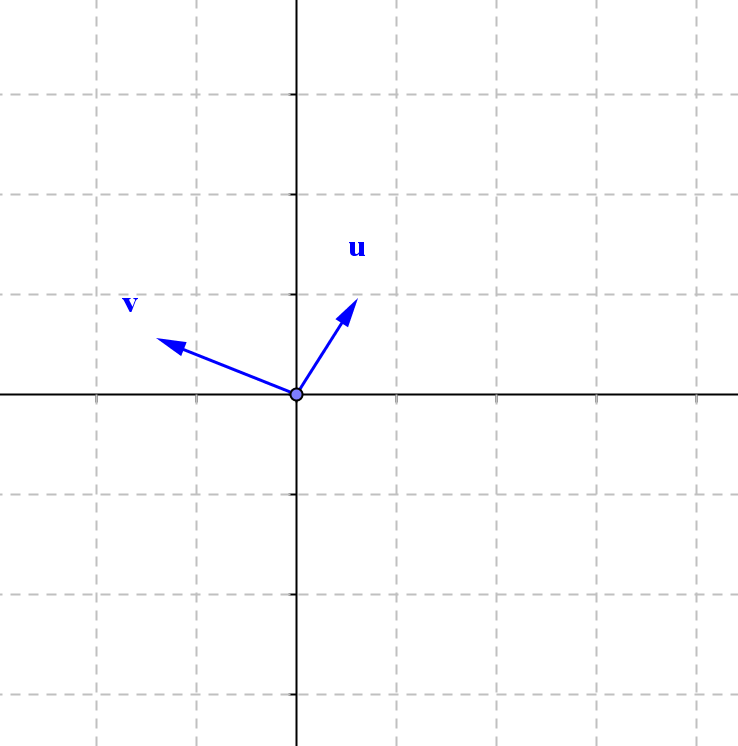
* 1. Explain how you add and subtract complex numbers.

Exercise 1

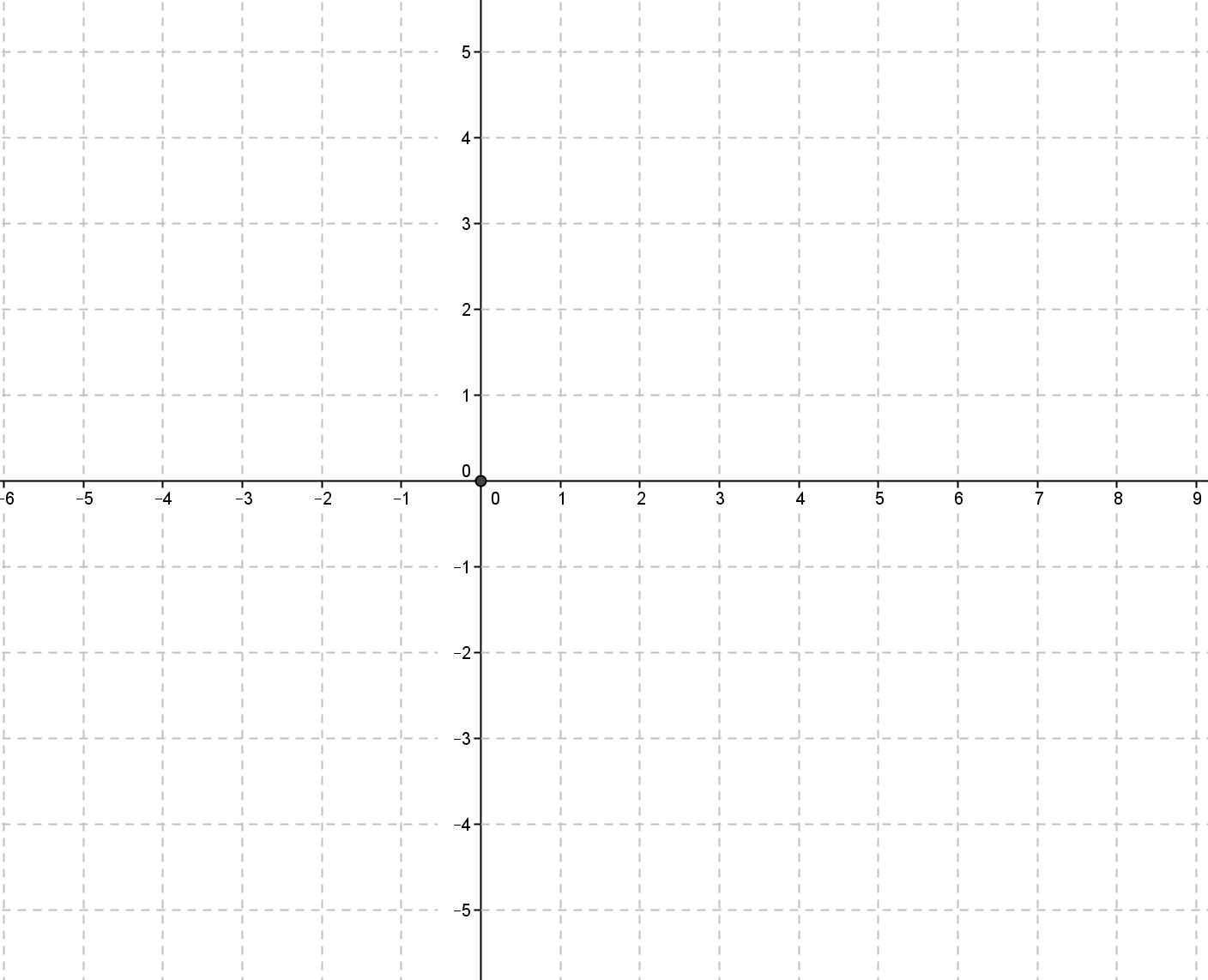
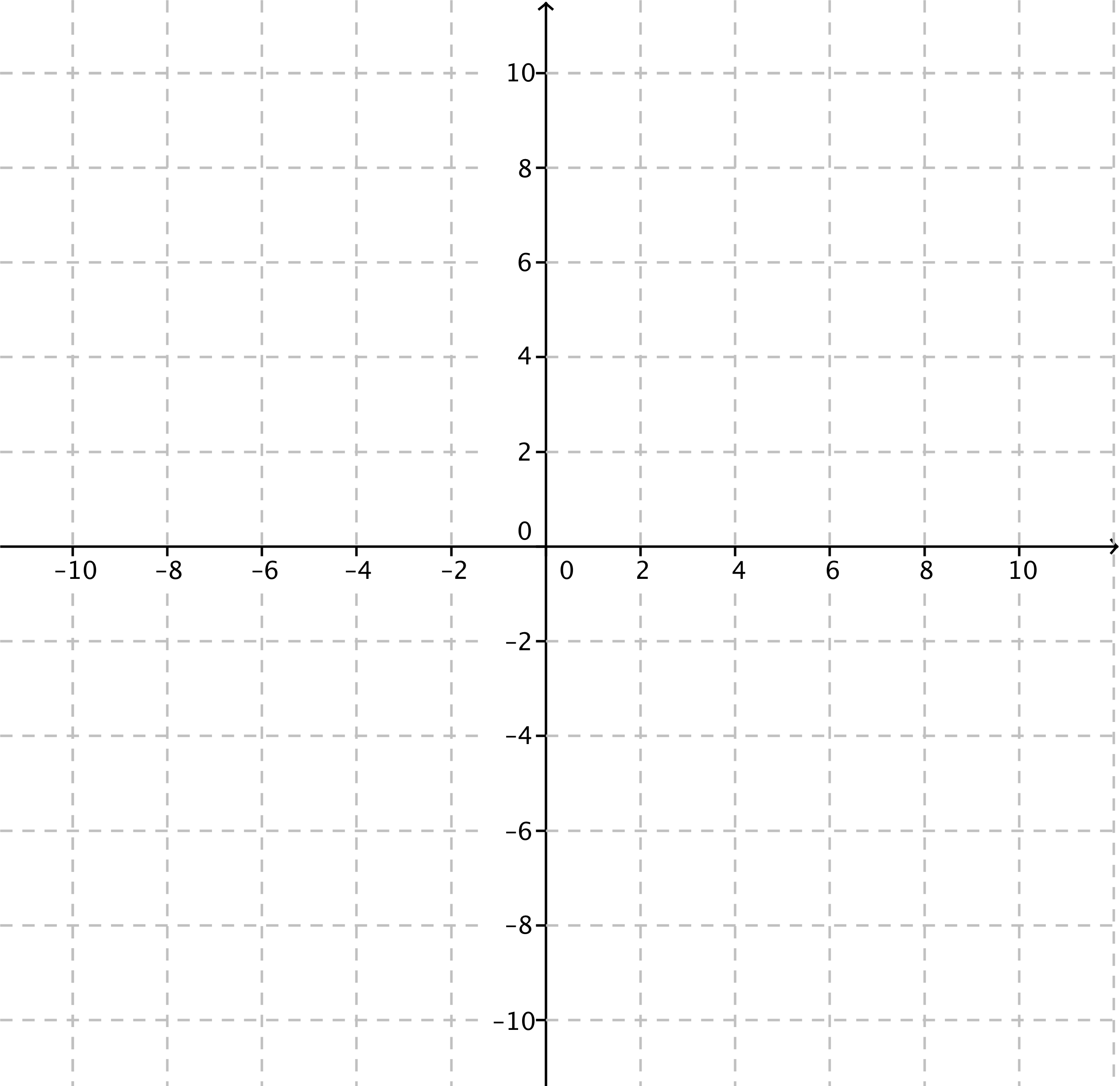
1. The length of the vector that represents is because .
   1. Find at least seven other complex numbers that can be represented as vectors that have length .
   2. Draw the vectors on the coordinate axes provided below.



* 1. What do you observe about all of these vectors?

1. In the Opening Exercises, we computed . Calculate this sum using vectors.
2. In the Opening Exercises, we also computed . Calculate this sum using vectors.
3. For the vectors and pictured below, draw the specified sum or difference on the coordinate axes provided.
   1. **
4. Find the sum of and geometrically.
5. Show that by representing the complex numbers as vectors.

Problem Set

1. Let and . Find the following. Express your answers in form.
   1. What is the length of the vector representing ?
   2. What is the length of the vector representing ?
2. Let , , and . Find the following. Express your answer in form, and represent the result in the plane.
   1. What is the length of the vector representing ?
   2. What is the length of the vector representing   
      ?
3. Find the sum of and geometrically.
4. Show that by representing the complex numbers as vectors.
5. Let , , and . Prove the following using algebra or by showing with vectors.
6. Let and .
   1. Draw vectors representing and on the same set of axes.
   2. What are the lengths of the vectors representing and ?
   3. Find a new vector, , such that is equal to divided by the length of the vector representing .
   4. Find , such that is equal to divided by the length of the vector representing .
   5. Draw vectors representing and on the same set of axes as part (a).
   6. What are the lengths of the vectors representing and ?
   7. Compare the vectors representing to and to . What do you notice?
   8. What is the value of times ?
   9. What does your answer to part (h) tell you about the relationship between and ?
7. Let .
   1. Let be represented by the vector in the direction of with length . How can you find ? What is the value of ?
   2. Let be the complex number that when multiplied by , the product is . What is the value of ?
   3. What number could we multiply by to get a product of ?
8. Let .
   1. Draw a picture representing .
   2. What is the value of ?