Lesson 3: Which Real Number Functions Define a Linear Transformation?

Classwork

Opening Exercises

Recall from the previous two lessons that a linear transformation is a function that satisfies two conditions:   
(1) and (2) . Here, refers to any real number, and and represent arbitrary elements in the domain of .

1. Let . Is a linear transformation? Explain why or why not.
2. Let . Is a linear transformation? Explain why or why not.

Problem Set

1. Suppose you have a linear transformation , where and .
   1. Use the addition property to compute , , , and .
   2. Find , , and . Show your work.
   3. Find , , and . Show your work.
   4. Find a formula for .
   5. Draw the graph of the function .
2. The symbol represents the set of integers, and so represents a function that takes integers as inputs and produces integers as outputs. Suppose that a function satisfies for all integers and . Is there necessarily an integer such that for all integer inputs ?
   1. Let . Compute and .
   2. Let be any positive integer. Compute .
   3. Now consider . Since , what can you conclude about ?
   4. Lastly, use the fact that to learn something about , where is any positive integer.
   5. Use your work above to prove that for every integer . Be sure to consider the fact that could be positive, negative, or .
3. In the following problems, be sure to consider all kinds of functions: polynomial, rational, trigonometric, exponential, logarithmic, etc.
   1. Give an example of a function that satisfies .
   2. Give an example of a function that satisfies .
   3. Give an example of a function that satisfies .