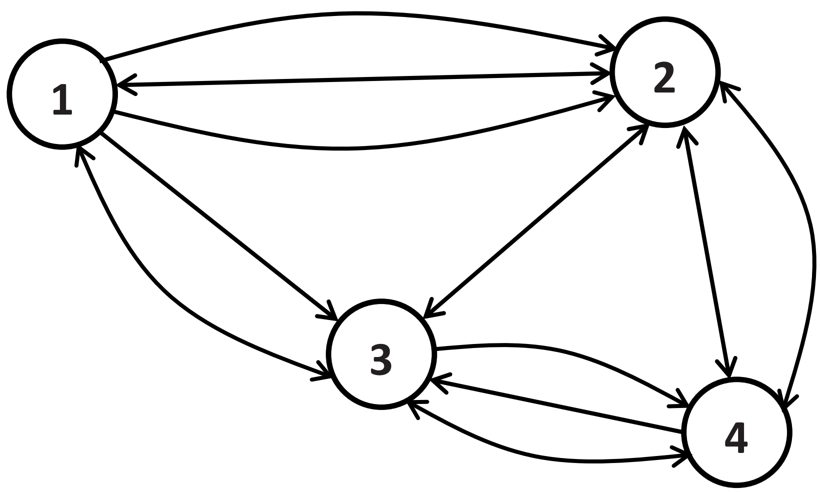
Name ____ Date ____________________

Lesson 1: Introduction to Networks

Exit Ticket

The following directed graph shows the major roads that connect four cities.



1. Create a matrix C that shows the direct routes connecting the four cities.

2. Use the matrix to determine how many ways are there to travel from City 1 to City 4 with one stop in City 2.

3. What is the meaning of $c_{2,3}$?

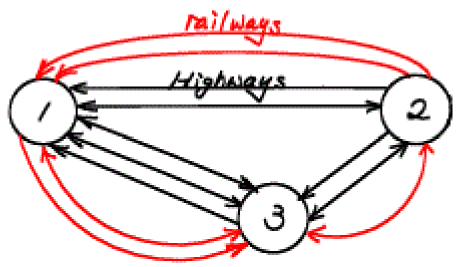
4. Write an expression that represents the total number of ways to travel between City 2 and City 3 without passing through the same city twice (you can travel through another city on the way from City 2 to City 3).

Name ____ Date ____________________

Lesson 2: Networks and Matrix Arithmetic

Exit Ticket

The diagram below represents a network of highways and railways between three cities. Highways are represented by black lines, and railways are represented by red lines.



1. Create matrix A that represents the number of major highways connecting the three cities and matrix B that represents the number of railways connecting the three cities.

2. Calculate and interpret the meaning of each matrix in this situation.
   a. $A + B$

   b. $3B$

3. Find $A - B$. Does the matrix $A - B$ have any meaning in this situation? Explain your reasoning.

Name ____ Date ____________________

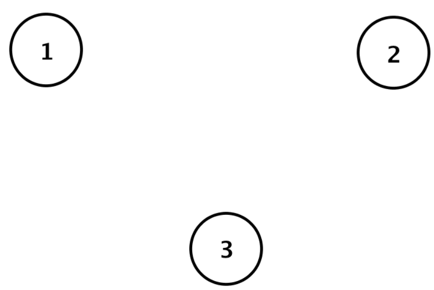
Lesson 3: Matrix Arithmetic in its Own Right

Exit Ticket

Matrix A represents the number of major highways connecting three cities. Matrix B represents the number of railways connecting the same three cities.

$$A = \begin{bmatrix} 0 & 3 & 0 \\ 2 & 0 & 2 \\ 1 & 1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

1. Draw a network diagram for the transportation network of highways and railways between these cities. Use solid lines for highways and dotted lines for railways.



2. Calculate and interpret the meaning of each matrix in this situation.
   a. $A \cdot B$

   b. $B \cdot A$

3. In this situation, why does it make sense that $A \cdot B \neq B \cdot A$?

Name ______________________________ Date ____________________

Lesson 4: Linear Transformations Review

Exit Ticket

1. In Module 1, we learned about linear transformations for any real-number functions. What are the conditions of a linear transformation? If a real-number function is a linear transformation, what is its form? What are the two characteristics of the function?

2. Describe the geometric effect of each mapping:
   a. $L(x) = 3x$

   b. $L(z) = (\sqrt{2} + \sqrt{2}i) \cdot z$

   c. $L(z) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$, where z is a complex number

   d. $L(z) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$, where z is a complex number

Name ______________________________ Date ______________

Lesson 5: Coordinates of Points in Space

Exit Ticket

1. Find the sum of the following, and plot the points and the resultant. Describe the geometric interpretation.

   a. $\begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

   b. $\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

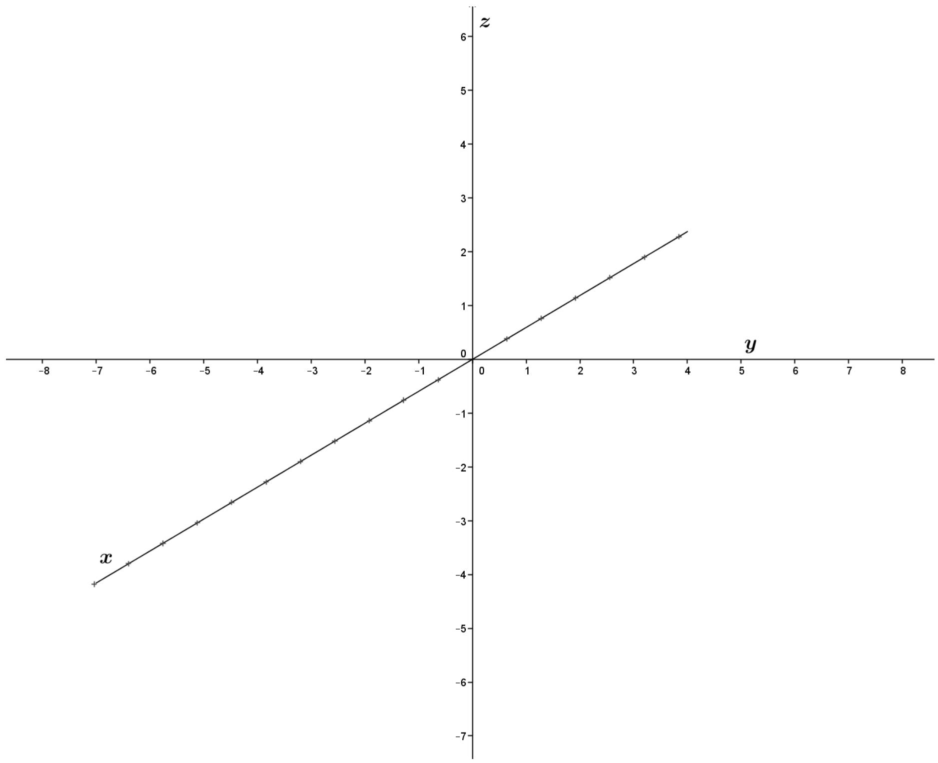
   c. $\begin{pmatrix} -2 \\ 4 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

   d. $-\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

2. Find the sum of the following.

   a. $\begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$

   b. $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$


z
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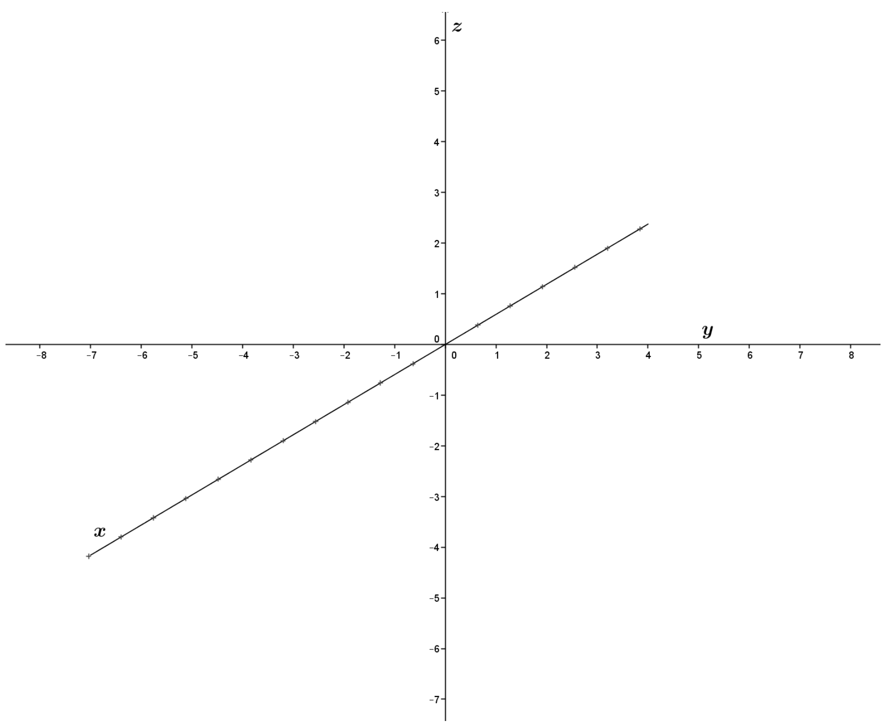
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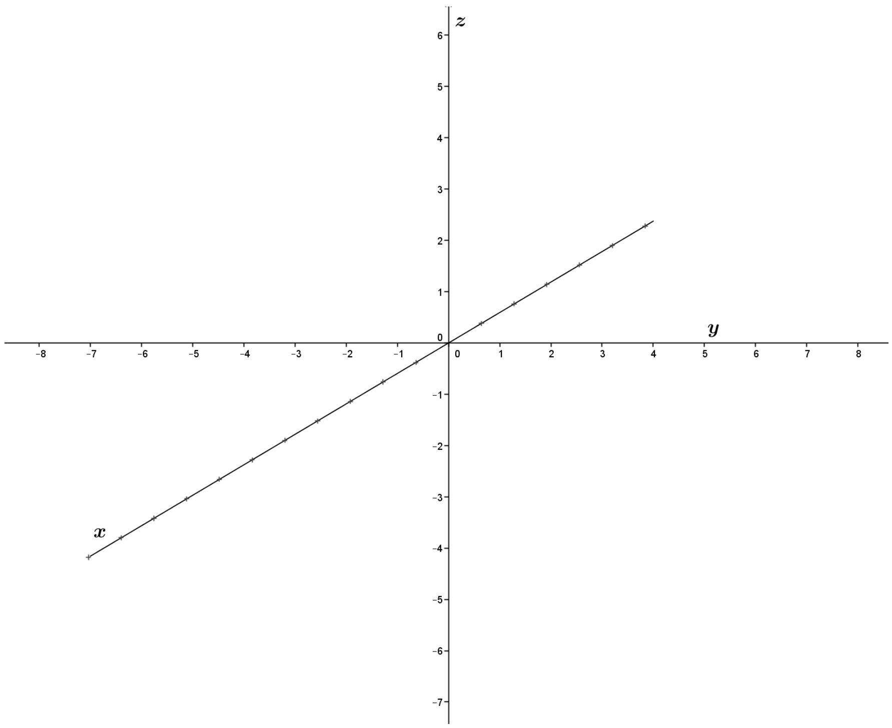
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-3
-2
-1
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1
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3
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6
7
8

Name ______________________________ Date ______________

Lesson 6: Linear Transformations as Matrices

Exit Ticket

1. Given $x = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $y = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$, $z = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, and $k = -2$.
   a. Verify the associative property holds: $x + (y + z) = (x + y) + z$.

   b. Verify the distributive property holds: $k(x + y) = kx + ky$.

2. Describe the geometric effect of the transformation on the 3×3 identity matrix given by the following matrices.
   a. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

   b. $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

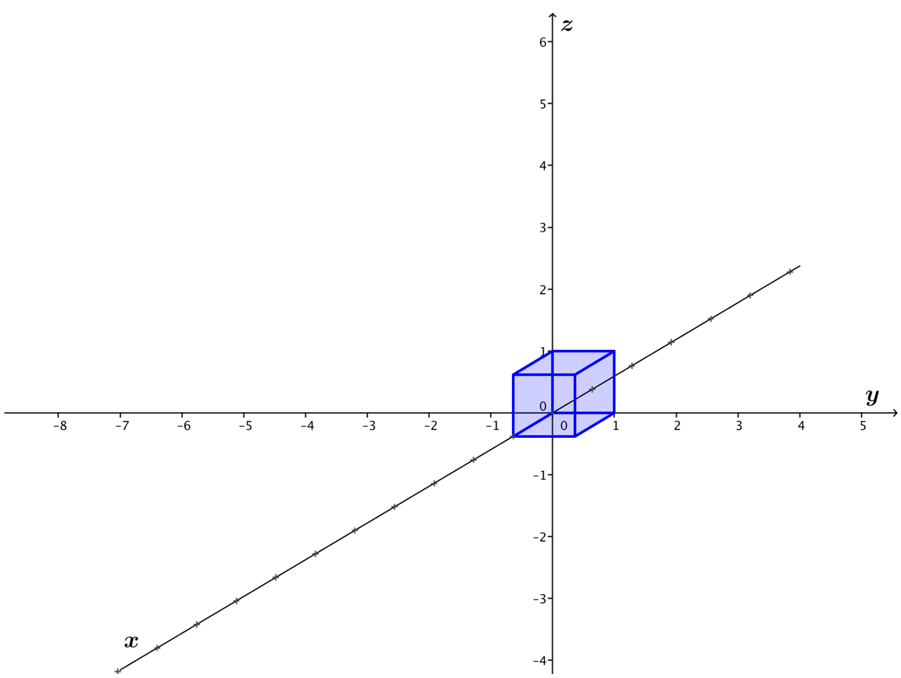
d. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Name ______________________________ Date ________________

Lesson 7: Linear Transformations Applied to Cubes

Exit Ticket

1. Sketch the image of the unit cube under the transformation $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ on the axes provided.



2. Does the transformation from Question 1 have an inverse? Explain how you know.

Name ____ Date ____________________

Lesson 8: Composition of Linear Transformations

Exit Ticket

Let A be the matrix representing a rotation about the origin $135°$ and B be the matrix representing a dilation of 6. Let $x = \begin{bmatrix} -1 \\ \frac{1}{2} \end{bmatrix}$.

a. Write down A and B.

b. Find the matrix representing a dilation of x by 6, followed by a rotation about the origin of $135°$.

c. Graph and label x, x after a dilation of 6, and x after both transformations have been applied.

Name ______________________________ Date ______________

Lesson 9: Composition of Linear Transformations

Exit Ticket

Let A be the matrix representing a rotation about the z-axis of $45°$ and B be the matrix representing a dilation of 2.

a. Write down A and B.

b. Let $x = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$. Find the matrix representing a dilation of x by 2 followed by a rotation about the z-axis of $45°$.

c. Do your best to sketch a picture of x, x after the first transformation, and x after both transformations. You may use technology to help you.

Name ____ Date ____________________

Lesson 10: Matrix Multiplication Is Not Commutative

Exit Ticket

1. Let A be the matrix representing a rotation about the origin by $60°$ and B be the matrix representing a reflection across the x-axis.

   a. Give two reasons why $AB \neq BA$.

   b. Let $x = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. Evaluate $A(Bx)$ and $B(Ax)$.

2. Write two matrices, A, B, that represent linear transformations where $A(Bx)$ and $B(Ax)$ where $x = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$. Explain why the products are the same.

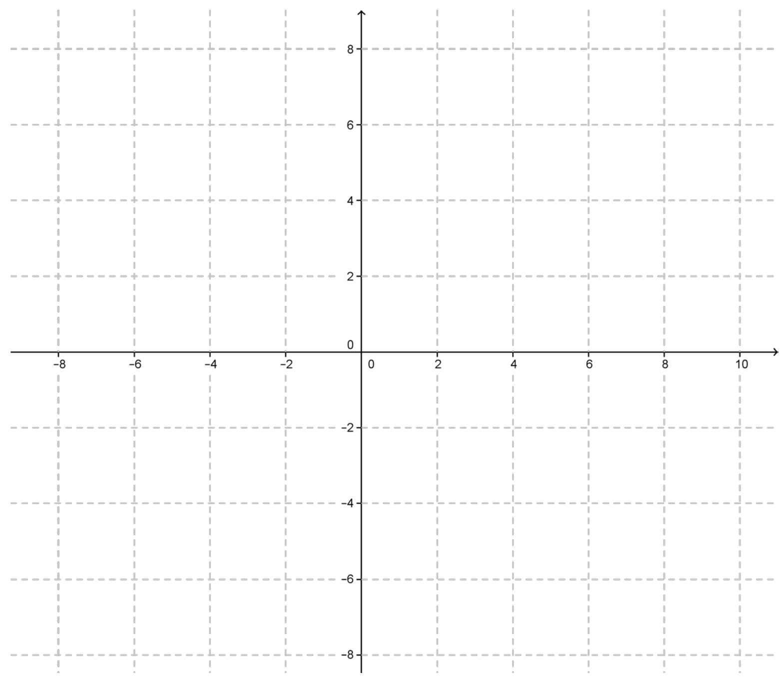
Name ____ Date ____________________

Lesson 11: Matrix Addition Is Commutative

Exit Ticket

1. Let $x = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$, $A = \begin{pmatrix} 2 & 2 \\ 2 & 0 \end{pmatrix}$, and $B = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$.

   a. Find and plot the points Ax, Bx, and $(A+B)x$ on the axes below.



   b. Show algebraically that matrix addition is commutative, $Ax + Bx = Bx + Ax$.

2. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $B = \begin{pmatrix} x & y \\ z & w \end{pmatrix}$. Prove $A + B = B + A$.

Name ______________________________ Date ______________

Lesson 12: Matrix Multiplication Is Distributive and Associative

Exit Ticket

In three-dimensional space, matrix A represents a $180°$ rotation about the y-axis, matrix B represents a reflection about the xz-plane, and matrix C represents a reflection about xy-plane. Answer the following:

a. Write matrices A, B, and C.

b. If $X = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$, compute $A(BX + CX)$.

c. What matrix operations are equivalent to $A(BX + CX)$? What property is shown?

d. Would $\big(A(BC)\big)X = \big((AB)C\big)X$? Why?

Name ____ Date ____________________

Lesson 13: Using Matrix Operations for Encryption

Exit Ticket

Morgan used matrix $E = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$ to encode the name of her favorite mathematician in the message

$$-32, 7, 14, 1, 52, 2, -13, -1.$$

a. How can you tell whether or not her message can be decoded?

b. Decode the message, or explain why the original message cannot be recovered.

Name ____ Date ______________________

1. Kyle wishes to expand his business and is entertaining four possible options. If he builds a new store he expects to make a profit of 9 million dollars if the market remains strong; however, if market growth declines, he could incur a loss of 5 million dollars. If Kyle invests in a franchise, he could profit 4 million dollars in a strong market but lose 3 million dollars in a declining market. If he modernizes his current facilities, he could profit 4 million dollars in a strong market but lose 2 million dollars in a declining one. If he sells his business, he will make a profit of 2 million dollars irrespective of the state of the market.

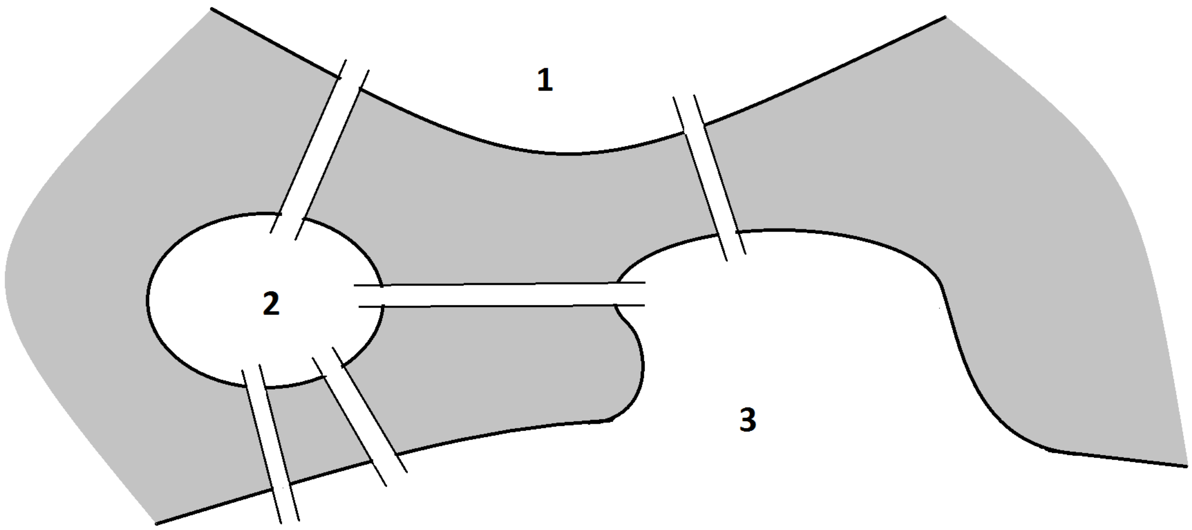
   a. Write down a 4×2 payoff matrix P summarizing the profits and losses Kyle could expect to see with all possible scenarios. (Record a loss as a profit in a negative amount.) Explain how to interpret your matrix.

   b. Kyle realized that all his figures need to be adjusted by 10% in magnitude due to inflation costs. What is the appropriate value of a real number λ so that the matrix λP represents a correctly adjusted payoff matrix? Explain your reasoning. Write down the new payoff matrix λP.

   c. Kyle is hoping to receive a cash donation of 1 million dollars. If he does, all the figures in his payoff matrix will increase by 1 million dollars.

      Write down a matrix Q so that if Kyle does receive this donation, his new payoff matrix is given by $Q + \lambda P$. Explain your thinking.

2. The following diagram shows a map of three land masses, numbered region 1, region 2, and region 3, connected via bridges over water. Each bridge can be traversed in either direction.



a. Write down a 3×3 matrix A with a_{ij}, for $i = 1, 2,$ or 3 and $j = 1, 2,$ or 3, equal to the number of ways to walk from region i to region j by crossing exactly one bridge. Notice that there are no paths that start and end in the same region crossing exactly one bridge.

b. Compute the matrix product A^2.

c. Show that there are 10 walking routes that start and end in region 2, crossing over water exactly twice. Assume each bridge, when crossed, is fully traversed to the next land mass.

d. How many walking routes are there from region 3 to region 2 that cross over water exactly three times? Again, assume each bridge is fully traversed to the next land mass.

e. If the number of bridges between each pair of land masses is doubled, how does the answer to part (d) change? That is, what would be the new count of routes from region 3 to region 2 that cross over water exactly three times?

3. Let $P = \begin{bmatrix} 3 & -5 \\ 5 & 3 \end{bmatrix}$ and $Q = \begin{bmatrix} -2 & 1 \\ -1 & -2 \end{bmatrix}$.

   a. Show the work needed and compute $2P - 3Q$.

   b. Show the work needed and compute PQ.

   c. Show that $P^2Q = PQP$.

4.

a. Show that if the matrix equation $(A + B)^2 = A^2 + 2AB + B^2$ holds for two square matrices A and B of the same dimension, then these two matrices commute under multiplication.

b. Give an example of a pair of 2×2 matrices A and B for which $(A + B)^2 \neq A^2 + 2AB + B^2$.

c. In general, does $AB = BA$? Explain.

d. In general, does $A(B + C) = AB + AC$? Explain.

e. In general, does $A(BC) = (AB)C$? Explain.

5. Let I be the 3×3 identity matrix and A the 3×3 zero matrix. Let the 3×1 column $x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ represent a point in three dimensional space. Also, set $P = \begin{bmatrix} 2 & 0 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix}$.

   a. Use examples to illustrate how matrix A plays the same role in matrix addition that the number 0 plays in real number addition. Include an explanation of this role in your response.

   b. Use examples to illustrate how matrix I plays the same role in matrix multiplication that the number 1 plays in real number multiplication. Include an explanation of this role in your response.

   c. What is the row 3, column 3 entry of $(AP + I)^2$? Explain how you obtain your answer.

d. Show that $(P-1)(P+1)$equals $P^2 - I$.

e. Show that Px is sure to be a point in the xz-plane in three-dimensional space.

f. Is there a 3×3 matrix Q, not necessarily the matrix inverse for P, for which $QPx = x$ for every 3×1 column x representing a point? Explain your answer.

g. Does the matrix P have a matrix inverse? Explain your answer.

h. What is the determinant of the matrix P?

6. What is the image of the point given by the 3×1 column matrix $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ when it is rotated $45°$ about the z-axis in the counterclockwise direction (as according to the orientation of the xy-plane) and then $180°$ about the y-axis?

Name ______________________________ Date ________________

Lesson 14: Solving Equations Involving Linear Transformations of the Coordinate Plane

Exit Ticket

In two-dimensional space, point x is rotated $180°$ to the point $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

a. Represent the transformation of point x using an equation in the format $Lx = b$.

b. Use inverse matrix operations to find the coordinates of x.

c. Verify that this solution makes sense geometrically.

Name ____ Date ____________________

Lesson 15: Solving Equations Involving Linear Transformations of the Coordinate Space

Exit Ticket

The lemonade sales at a baseball game were described as follows:

The number of small lemonades purchased was the number of mediums sold plus double the number of larges sold.

The total number of all sizes sold was 70.

One and a half times the number of smalls purchased plus twice the number of mediums sold was 100.

Use a system of equations and its matrix representation to determine the number of small, medium, and large lemonades sold.

Name ____ Date ____________________

Lesson 16: Solving General Systems of Linear Equations

Exit Ticket

1. Anabelle, Bryan, and Carl are playing a game using sticks of gum. For each round of the game, Anabelle gives half of her sticks of gum to Bryan and one-fourth to Carl. Bryan gives one-third of his sticks to Anabelle and keeps the rest. Carl gives 40 percent of his sticks of gum to Anabelle and 10 percent to Bryan. Sticks of gum can be cut into fractions when necessary.

   a. After one round of the game, the players count their sticks of gum. Anabelle has 525 sticks of gum, Bryan has 600, and Carl has 450. How many sticks of gum will each player have after 2 more rounds of the game? Use a matrix equation to represent the situation, and explain your answer in context.

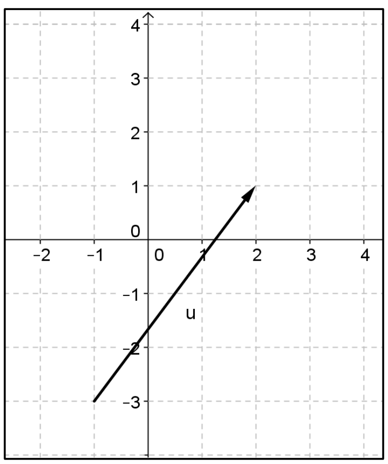
   b. How many sticks of gum did each player have at the start of the game? Use a matrix equation to represent the situation, and explain your answer in context.

Name ______________________________ Date ______________

Lesson 17: Vectors in the Coordinate Plane

Exit Ticket

1. Vector $\mathbf{v} = \langle 3,4 \rangle$, and the vector $\mathbf{u}$ is represented by the arrow shown below. How are the vectors the same? How are they different?



2. Let $\mathbf{u} = \langle 1,5 \rangle$ and $\mathbf{v} = \langle 3,-2 \rangle$. Write each vector in component form and draw an arrow to represent the vector.
   a. $\mathbf{u} + \mathbf{v}$

   b. $\mathbf{u} - \mathbf{v}$

   c. $2\mathbf{u} + 3\mathbf{v}$

3. For $\mathbf{u} = \langle 1,5 \rangle$ and $\mathbf{v} = \langle 3,-2 \rangle$ as in 2(a), what is the magnitude of $\mathbf{u} + \mathbf{v}$?

Name ____ Date ____________________

Lesson 18: Vectors and Translation Maps

Exit Ticket

1. Given the vector $\mathbf{v} = \langle 2, -1 \rangle$, find the image of the line $3x - 2y = 2$ under the translation map $T_{\mathbf{v}}$. Graph the original line and its image, and explain the geometric effect of the map $T_{\mathbf{v}}$.

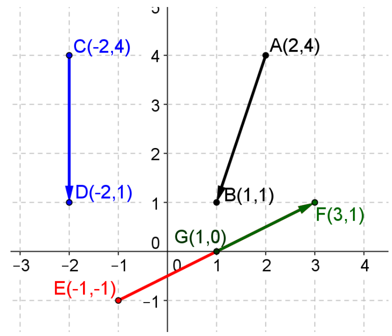
2. Given the vector $\mathbf{v} = \langle -1, 2 \rangle$, find the image of the circle $(x-2)^2 + (y+1)^2 = 4$ under the translation map $T_{\mathbf{v}}$. Graph the original circle and its image, and then explain the geometric effect of the map $T_{\mathbf{v}}$.

Name ______________________________ Date ______________

Lesson 19: Directed Line Segments and Vectors

Exit Ticket

1. Consider vectors with their initial and terminal points as shown below. Find the components of the specified vectors and their magnitudes.



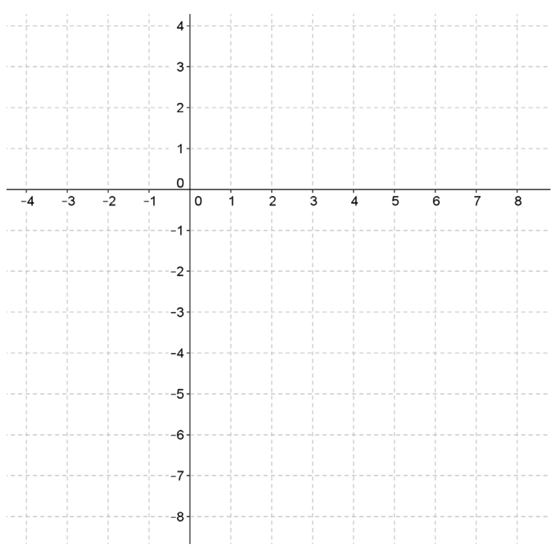
   a. $\mathbf{u} = \overrightarrow{EF}$ and $\|\mathbf{u}\|$

   b. $\mathbf{v} = \overrightarrow{AB}$ and $\|\mathbf{v}\|$

   c. $\mathbf{w} = \overrightarrow{CD}$ and $\|\mathbf{w}\|$

   d. $\mathbf{t} = \overrightarrow{GF}$ and $\|\mathbf{t}\|$

2. For vectors $\mathbf{u}$ and $\mathbf{v}$ as in Question 1, explain how to find $\mathbf{u} + \mathbf{v}$ using the parallelogram rule. Support your answer graphically below.

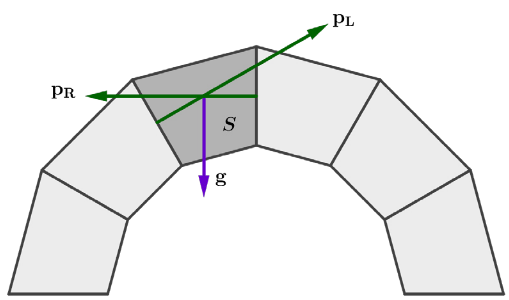


Name ____ Date ____________________

Lesson 20: Vectors and Stone Bridges

Exit Ticket

We saw in the lesson that the forces acting on a stone in a stable arch must sum to zero since the stones do not move. Now, we will consider the upper-left stone in a stable arch made of six stones. We will denote this stone by S. In the image below, $\mathbf{p_L}$ represents the force acting on stone S from the stone on the left. Vector $\mathbf{p_R}$ represents the force acting on stone S from the stone on the right. Vector $\mathbf{g}$ represents the downward force of gravity.



a. Describe the directions of vectors $\mathbf{g}$, $\mathbf{p_L}$, and $\mathbf{p_R}$ in terms of rotation from the positive x-axis by θ degrees, for $-180 < \theta < 180$.

b. Suppose that vector $\mathbf{g}$ has a magnitude of 1. Find the magnitude of vectors $\mathbf{p_L}$ and $\mathbf{p_R}$.

c. Write vectors $\mathbf{g}$, $\mathbf{p_L}$, and $\mathbf{p_R}$ in magnitude and direction form.

Name ______________________________ Date ______________

Lesson 21: Vectors and the Equation of a Line

Exit Ticket

1. Find parametric equations for the line in the plane given by $y = 2x + 3$.

2. Do $y = 7 - x$ and $x(t) = -1$ and $y(t) = 1 + 7t$ represent the same line? Explain why or why not.

3. Find parametric equations for the line in space that passes through point $(1,0,4)$ with direction vector $\vec{\mathbf{v}} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$.

Name ____ Date ____________________

Lesson 22: Linear Transformations of Lines

Exit Ticket

1. Consider points $P(2,1)$ and $Q(2,5)$. Find parametric equations that describe points on the line segment $\overline{PQ}$.

2. Suppose that points $P(2,1)$ and $Q(2,5)$ are transformed under the linear transformation $L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$. Find parametric equations that describe the image of line $\overleftrightarrow{PQ}$ under this transformation.

Name ____ Date ____________________

Lesson 23: Why Are Vectors Useful?

Exit Ticket

A hailstone is traveling through the sky. Its position $\begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix}$ in meters is given by $\begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2160 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \\ -9 \end{pmatrix} t$ where t is the time in seconds since the hailstone began being tracked.

a. If $x(t)$ represents an east-west location, how quickly is the hailstone moving to the east?

b. If $y(t)$ represents a north-south location, how quickly is the hailstone moving to the south?

c. What could be causing the east-west and north-south velocities for the hailstone?

d. If $z(t)$ represents the height of the hailstone, how quickly is the hailstone falling?

e. At what location will the hailstone hit the ground (assume $z = 0$ is the ground)? How long will this take?

f. What is the overall speed of the hailstone? To the nearest meter, how far did the hailstone travel from $t = 0$ to the time it took to hit the ground?

Name ____ Date ________________

Lesson 24: Why Are Vectors Useful?

Exit Ticket

1. Consider the system of equations $\begin{cases} y = 5x + 2 \\ y = 3x \end{cases}$, and perform the following operations on an arbitrary point $\begin{pmatrix} x \\ y \end{pmatrix}$:

   a. Rotate around the origin by θ.

   b. Translate by the opposite of the solution to the system.

   c. Apply a dilation of $2/3$.

2. What effect does each of the transformations in Problem 1 have on the solution of the system and on the origin?

Name ____ Date ____________________

Lesson 25: First-Person Computer Games

Exit Ticket

In a computer game, the camera eye is at the origin, and the tip of a dog's nose has coordinates $\begin{pmatrix} 2 \\ 10 \\ 3 \end{pmatrix}$. If the computer screen represents the plane $y = 1$, determine the coordinates of the projected point that represents the tip of the dog's nose.

Name ____ Date ____________________

Lesson 26: Projecting a 3-D Object onto a 2-D Plane

Exit Ticket

1. Consider the plane defined by $z = 2$ and the points, $x = \begin{pmatrix} 3 \\ 6 \\ 8 \end{pmatrix}$ and $y = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}$.

   a. Find the projections of x and y onto the plane $z = 2$ if the eye is placed at the origin.

   b. Consider $w = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$; does it make sense to find the projection of w onto $z = 2$? Explain.

2. Consider an object located at $\begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$ and rotating around the z-axis. At what θ value will the object be out of sight of the plane $y = 1$?

Name ____ Date ____________________

Lesson 27: Designing Your Own Game

Exit Ticket

Consider the following set of code for the ALICE program featuring a bluebird whose center is located at $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

this.bluebird resize 2.0
this.bluebird resizeHeight 0.5
this.bluebird turn RIGHT 0.25
this.bluebird move FORWARD 1.0

For an arbitrary point x on this bluebird, write the four matrices that represent the code above, and state where the point ends after the program runs.

Name ____ Date ____________________

1.

a. Find values for a, b, c, d, and e so that the following matrix product equals the 3×3 identity matrix. Explain how you obtained these values.

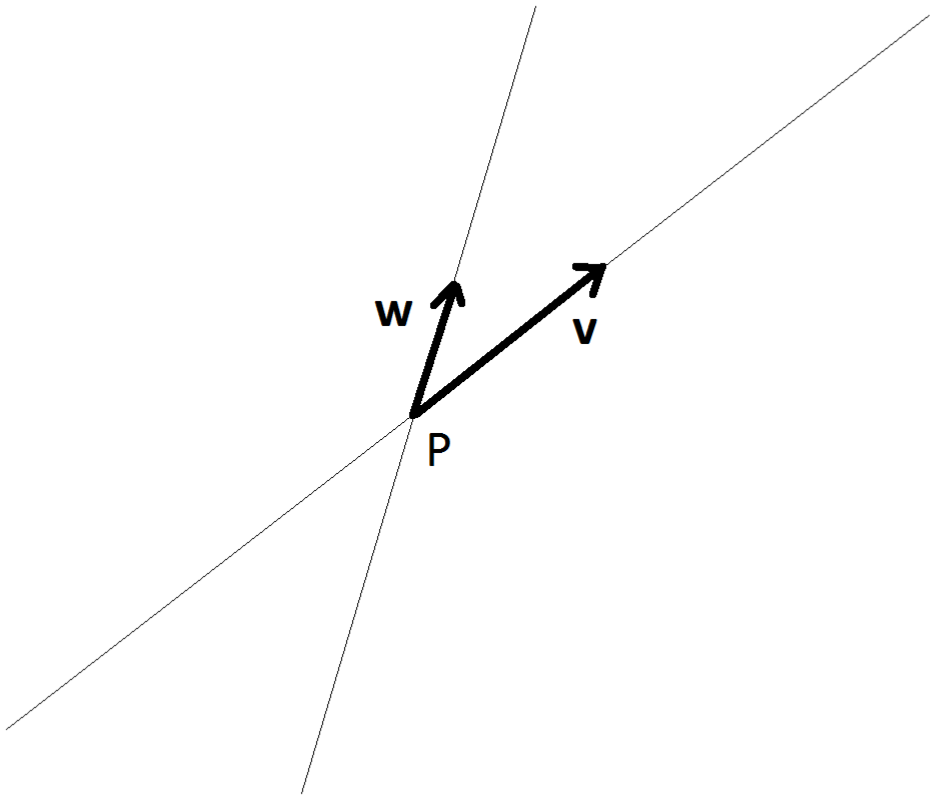
$$\begin{bmatrix} a & -3 & 5 \\ c & c & 1 \\ 5 & b & -4 \end{bmatrix}\begin{bmatrix} 1 & b & d \\ 1 & c & e \\ 2 & b & b \end{bmatrix}$$

b. Represent the following system of linear equations as a single matrix equation of the form $Ax = b$, where A is a 3×3 matrix, and x and b are 3×1 column matrices.

$$x + 3y + 2z = 8$$
$$x - y + z = -2$$
$$2x + 3y + 3z = 7$$

c. Solve the system of three linear equations given in part (b).

2. The following diagram shows two two-dimensional vectors $\mathbf{v}$ and $\mathbf{w}$ in the place positioned to both have endpoint at point P.



a. On the diagram, make reasonably accurate sketches of the following vectors, again each with endpoint at P. Be sure to label your vectors on the diagram.
   i. $2\mathbf{v}$
   ii. $-\mathbf{w}$
   iii. $\mathbf{v} + 3\mathbf{w}$
   iv. $\mathbf{w} - 2\mathbf{v}$
   v. $\frac{1}{2}\mathbf{v}$

Vector $\mathbf{v}$ has magnitude 5 units, $\mathbf{w}$ has magnitude 3, and the acute angle between them is $45°$.

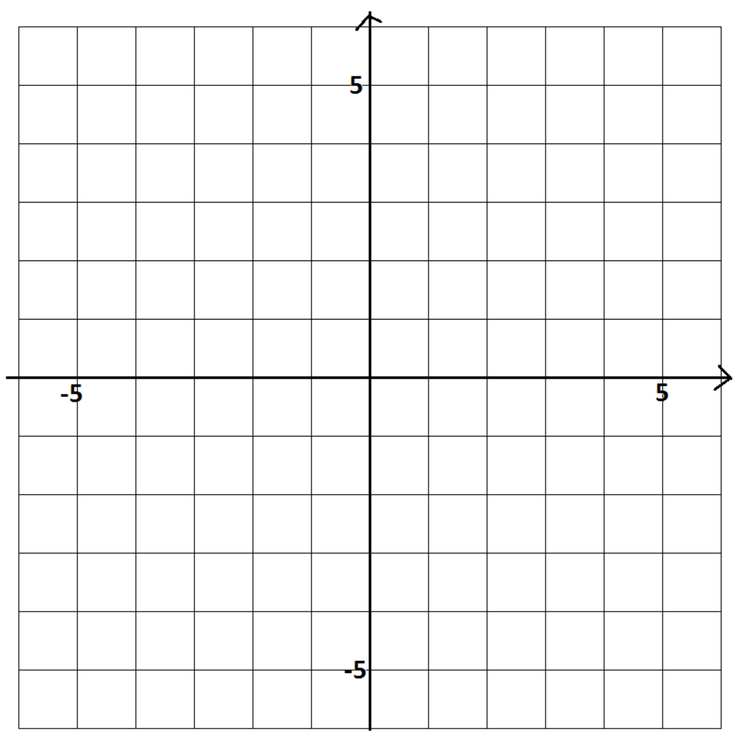
b. What is the magnitude of the scalar multiple $-5\mathbf{v}$?

c. What is the measure of the smallest angle between $-5\mathbf{v}$ and $3\mathbf{w}$ if these two vectors are placed to have a common endpoint?

3. Consider the two-dimensional vectors $\mathbf{v} = \langle 2, 3\rangle$ and $\mathbf{w} = \langle -2, -1\rangle$.

a. What are the components of each of the vectors $\mathbf{v} + \mathbf{w}$ and $\mathbf{v} - \mathbf{w}$?

b. On the following diagram, draw representatives of each of the vectors $\mathbf{v}$, $\mathbf{w}$, and $\mathbf{v} + \mathbf{w}$, each with endpoint at the origin.



c. The representatives for the vectors $\mathbf{v}$ and $\mathbf{w}$ you drew form two sides of a parallelogram, with the vector $\mathbf{v}+\mathbf{w}$ corresponding to one diagonal of the parallelogram. What vector, directed from the third quadrant to the first quadrant, is represented by the other diagonal of the parallelogram? Express your answer solely in terms of $\mathbf{v}$ and $\mathbf{w}$, and also give the coordinates of this vector.

d. Show that the magnitude of the vector $\mathbf{v}+\mathbf{w}$ does not equal the sum of the magnitudes of $\mathbf{v}$ and of $\mathbf{w}$.

e. Give an example of a non-zero vector $\mathbf{u}$ such that $\|\mathbf{v}+\mathbf{u}\|$ does equal $\|\mathbf{v}\|+\|\mathbf{u}\|$.

f. Which of the following three vectors has the greatest magnitude: $\mathbf{v}+(-\mathbf{w})$, $\mathbf{w}-\mathbf{v}$, or $(-\mathbf{v})-(-\mathbf{w})$?

g. Give the components of a vector one-quarter the magnitude of vector $\mathbf{v}$ and with direction opposite the direction of $\mathbf{v}$.

4. Vector $\mathbf{a}$ points true north and has magnitude 7 units. Vector $\mathbf{b}$ points $30°$ east of true north. What should the magnitude of $\mathbf{b}$ be so that $\mathbf{b} - \mathbf{a}$ points directly east?

   State the magnitude and direction of $\mathbf{b} - \mathbf{a}$.

5. Consider the three points $A = (10, -3, 5)$, $B = (0, 2, 4)$, and $C = (2, 1, 0)$ in three-dimensional space. Let M be the midpoint of $\overline{AB}$ and N be the midpoint of $\overline{AC}$.

   a. Write down the components of the three vectors $\overrightarrow{AB}, \overrightarrow{BC}$, and $\overrightarrow{CA}$, and verify through arithmetic that their sum is zero. Also, explain why geometrically we expect this to be the case.

   b. Write down the components of the vector $\overrightarrow{MN}$. Show that it is parallel to the vector $\overrightarrow{BC}$ and half its magnitude.

   Let $G = (4, 0, 3)$.

   c. What is the value of the ratio $\frac{\|\overrightarrow{MG}\|}{\|\overrightarrow{MC}\|}$?

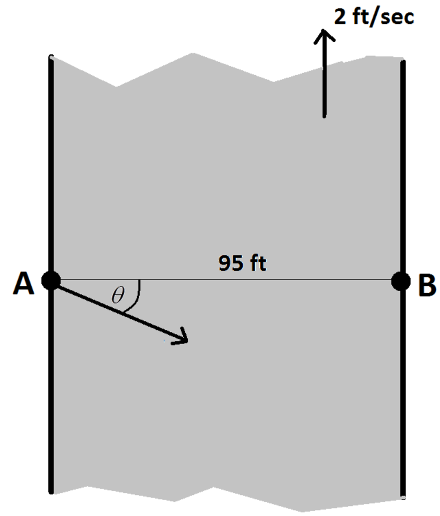
   d. Show that the point G lies on the line connecting M and C. Show that G also lies on the line connecting N and B.

6. A section of a river, with parallel banks 95 ft. apart, runs true north with a current of 2 ft/sec. Lashana, an expert swimmer, wishes to swim from point A on the west bank to the point B directly opposite it. In still water she swims at an average speed of 3 ft/sec.

   The diagram to the right illustrates the situation.

   To counteract the current, Lashana realizes that she is to swim at some angle θ to the east/west direction as shown.

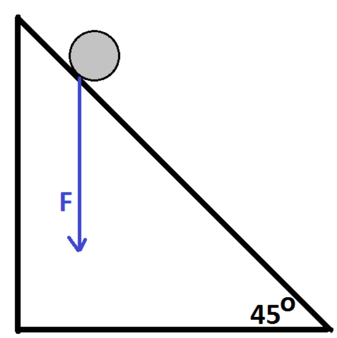
   With the simplifying assumptions that Lashana's swimming speed will be a constant 3 ft/sec and that the current of the water is a uniform 2 ft/sec flow northward throughout all regions of the river (we will ignore the effects of drag at the river banks, for example), at what angle θ to east/west direction should Lashana swim in order to reach the opposite bank precisely at point B? How long will her swim take?



   a. What is the shape of Lashana's swimming path according to an observer standing on the bank watching her swim? Explain your answer in terms of vectors.

b. If the current near the banks of the river is significantly less than 2 $\mathrm{ft/sec}$, and Lashana swims at a constant speed of 3 $\mathrm{ft/sec}$ at the constant angle θ to the east/west direction as calculated in part (a), will Lashana reach a point different from B on the opposite bank? If so, will she land just north or just south of B? Explain your answer.

7. A 5 kg ball experiences a force due to gravity $\vec{F}$ of magnitude 49 Newtons directed vertically downwards. If this ball is placed on a ramped tilted at an angle of $45°$, what is the magnitude of the component of this force, in Newtons, on the ball directed $45°$ towards the bottom of the ramp? (Assume the ball is of sufficiently small radius that is reasonable to assume that all forces are acting at the point of contact of the ball with the ramp.)



8. Let A be the point $(1, 1, -3)$ and B be the point $(-2, 1, -1)$ in three-dimensional space.

A particle moves along the straight line through A and B at uniform speed in such a way that at time $t = 0$ seconds the particle is at A, and at $t = 1$ second the particle is at B. Let $P(t)$ be the location of the particle at time t (so, $P(0) = A$ and $P(1) = B$).

a. Find the coordinates of the point $P(t)$ each in terms of t.

b. Give a geometric interpretation of the point $P(0.5)$.

Let L be the linear transformation represented by the 3×3 matrix $\begin{bmatrix} 2 & 0 & 1 \\ 1 & 3 & 0 \\ 0 & 1 & 1 \end{bmatrix}$, and let $A' = LA$ and $B' = LB$ be the images of the points A and B, respectively, under L.

c. Find the coordinates of A' and B'.

A second particle moves through three-dimensional space. Its position at time t is given by $L\left(P(t)\right)$, the image of the location of the first particle under the transformation L.

d. Where is the second particle at times $t = 0$ and $t = 1$? Briefly explain your reasoning.

e. Prove that second the particle is also moving along a straight line path at uniform speed.