Lesson 10: The Volume of Prisms and Cylinders and Cavalieri’s Principle

Classwork

Opening Exercise

The bases of the following triangular prism $T$ and rectangular prism $R$ lie in the same plane. A plane that is parallel to the bases and also a distance $3$ from the bottom base intersects both solids and creates cross-sections $T'$ and $R'$.

1. Find $Area(T^{'})$.
2. Find $Area(R^{'})$.
3. Find $Vol\left(T\right)$.
4. Find $Vol\left(R\right)$.
5. If a height other than $3$ were chosen for the cross-section, would the cross-sectional area of either solid change?

Discussion

Figure 1

 

Example 1



Example 2

**Principle of parallel slices in the plane:** If two planar figures of equal altitude have identical cross-sectional lengths at each height, then the regions of the figures have the same area.

Figure 2

Example

* 1. The following triangles have equal areas: $Area\left(△ABC\right)=Area\left(△A^{'}B^{'}C^{'}\right)=15 units^{2}$. The distance between $\overleftrightarrow{DE}$ and $\overleftrightarrow{CC'}$ is $3.$ Find the lengths $\overbar{DE}$ and $\overbar{D'E'}$.



* 1. Joey says that if two figures have the same height and the same area, then their cross-sectional lengths at each height will be the same. Give an example to show that Joey’s theory is incorrect.



Discussion

Figure 3

**Cavalieri’s principle:**  Given two solids that are included between two parallel planes, if every plane parallel to the two planes intersects both solids in cross-sections of equal area, then the volumes of the two solids are equal.

Figure 4

Figure 5

Figure 6

Problem Set

Lesson Summary

**Principle of parallel slices in the plane:** If two planar figures of equal altitude have identical cross-sectional lengths at each height, then the regions of the figures have the same area.

**Cavalieri’s principle:**  Given two solids that are included between two parallel planes, if every plane parallel to the two planes intersects both solids in cross-sections of equal area, then the volumes of the two solids are equal.

1. Use the principle of parallel slices to explain the area formula for a parallelogram.
2. Use the principle of parallel slices to show that the three triangles shown below all have the same area.

Figure 1 Figure 2 Figure 3

1. An oblique prism has a rectangular base that is $16 in.$ $×9 in$. A hole in the prism is also the shape of an oblique prism with a rectangular base that is $3 in.$ wide and $6 in.$ long, and the prism’s height is $9 in.$ (as shown in the diagram). Find the volume of the remaining solid.



1. An oblique circular cylinder has height $5$ and volume $45π$. Find the radius of the circular base.



1. A right circular cone and a solid hemisphere share the same base. The vertex of the cone lies on the hemisphere. Removing the cone from the solid hemisphere forms a solid. Draw a picture, and describe the cross-sections of this solid that are parallel to the base.
2. Use Cavalieri’s principle to explain why a circular cylinder with a base of radius $5$ and a height of $10$ has the same volume as a square prism whose base is a square with edge length $5\sqrt{π}$ and whose height is also $10$.