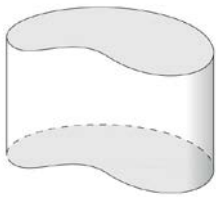


Lesson 7: General Pyramids and Cones and Their Cross-Sections

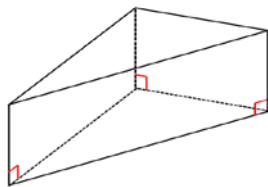
Classwork

Opening Exercise

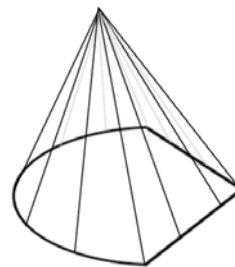
Group the following images by shared properties. What defines each of the groups you have made?



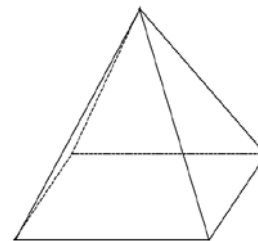
1



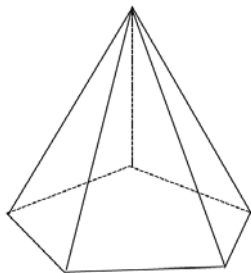
2



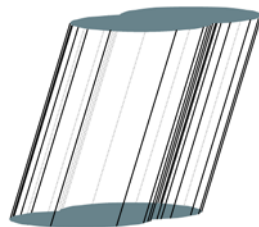
3



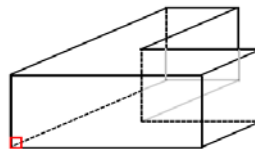
4



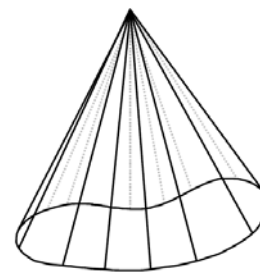
5



6



7



8

RECTANGULAR PYRAMID: Given a rectangular region B in a plane E and a point V not in E , the *rectangular pyramid* with base B and vertex V is the collection of all segments \overline{VP} for any point P in B .

GENERAL CONE: Let B be a region in a plane E and V be a point not in E . The *cone* with base B and vertex V is the union of all segments \overline{VP} for all points P in B (See Figures 1 and 2).

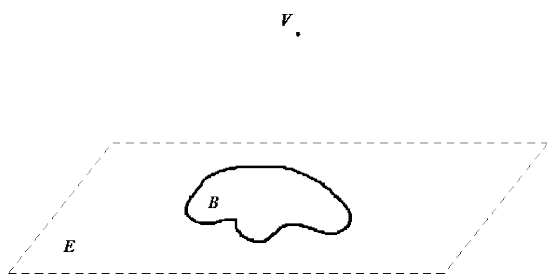


Figure 1

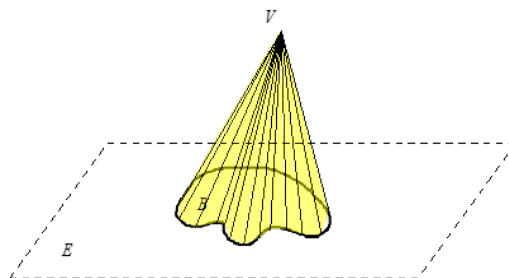


Figure 2

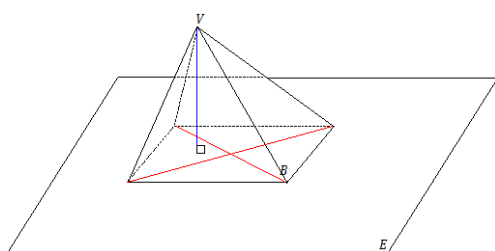


Figure 3

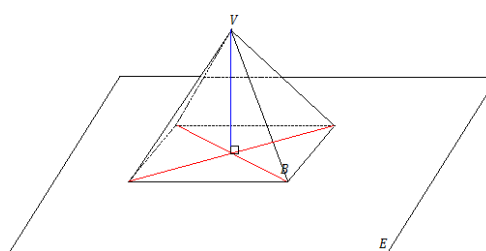


Figure 4

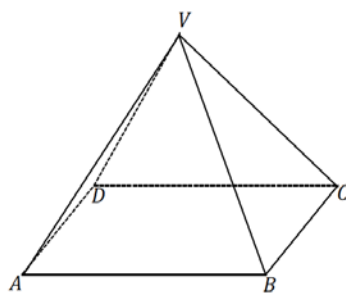


Figure 5

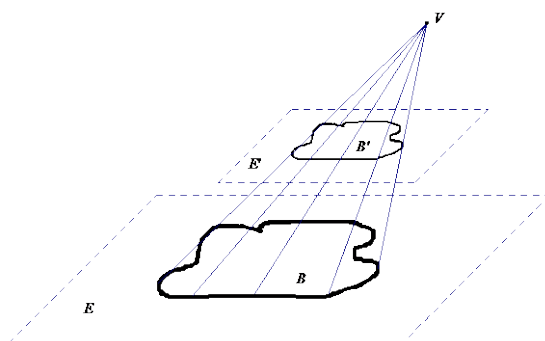
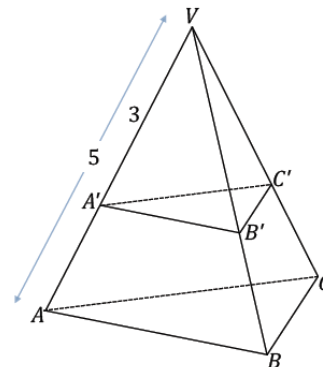


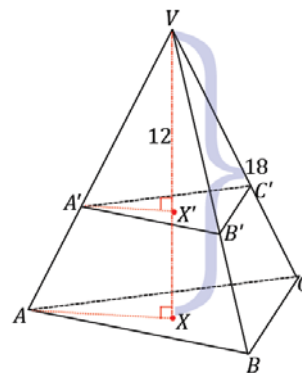
Figure 6

Example 1

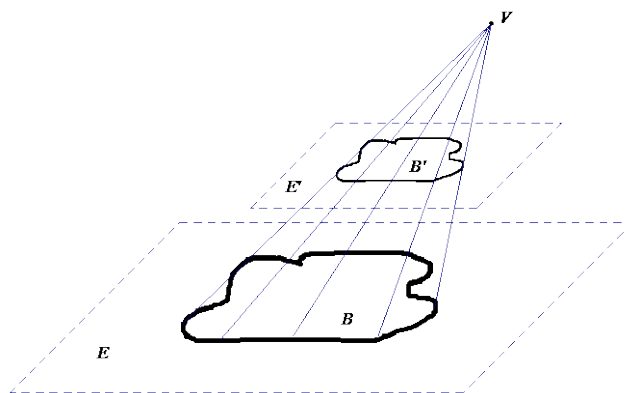
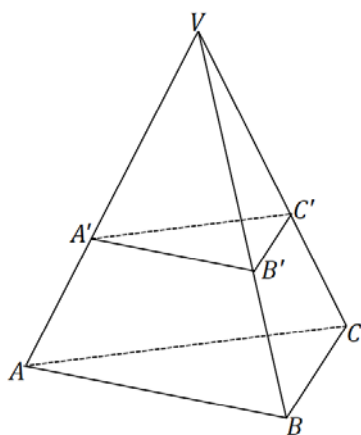
In the following triangular pyramid, a plane passes through the pyramid so that it is parallel to the base and results in the cross-section $\triangle A'B'C'$. If the area of $\triangle ABC$ is 25 mm^2 , what is the area of $\triangle A'B'C'$?

**Example 2**

In the following triangular pyramid, a plane passes through the pyramid so that it is parallel to the base and results in the cross-section $\triangle A'B'C'$. The altitude from V is drawn; the intersection of the altitude with the base is X , and the intersection of the altitude with the cross-section is X' . If the distance from X to V is 18 mm, the distance from X' to V is 12 mm, and the area of $\triangle A'B'C'$ is 28 mm^2 , what is the area of $\triangle ABC$?



Extension



Exercise 1

The area of the base of a cone is 16, and the height is 10. Find the area of a cross-section that is distance 5 from the vertex.

Example 3

GENERAL CONE CROSS-SECTION THEOREM: If two general cones have the same base area and the same height, then cross-sections for the general cones the same distance from the vertex have the same area.

State the theorem in your own words.

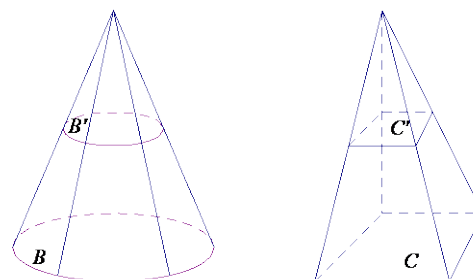
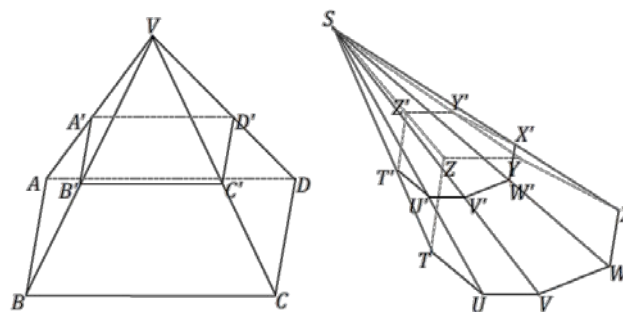


Figure 8

Use the space below to prove the *general cone cross-section theorem*.

Exercise 2

The following pyramids have equal altitudes, and both bases are equal in area and are coplanar. Both pyramids' cross-sections are also coplanar. If $BC = 3\sqrt{2}$ and $B'C' = 2\sqrt{3}$, and the area of $TUVWXYZ$ is 30 units², what is the area of cross-section $A'B'C'D'$?



Lesson Summary

CONE: Let B be a region in a plane E and V be a point not in E . The *cone with base B and vertex V* is the union of all segments \overline{VP} for all points P in B .

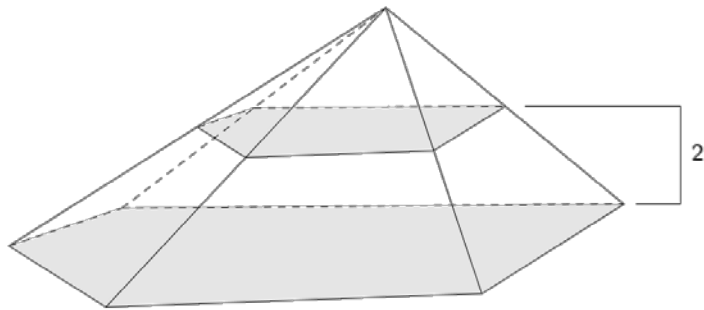
If the base is a polygonal region, then the *cone* is usually called a *pyramid*.

RECTANGULAR PYRAMID: Given a rectangular region B in a plane E and a point V not in E , the *rectangular pyramid with base B and vertex V* is the union of all segments \overline{VP} for points P in B .

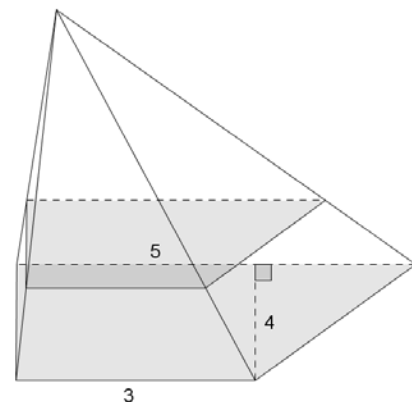
LATERAL EDGE AND FACE OF A PYRAMID: Suppose the base B of a pyramid with vertex V is a polygonal region and P_i is a vertex of B . The segment $\overline{P_iV}$ is called a *lateral edge* of the pyramid. If $\overline{P_iP_{i+1}}$ is a base edge of the base B (a side of B), and F is the union of all segments \overline{PV} for P in $\overline{P_iP_{i+1}}$, then F is called a *lateral face* of the pyramid. It can be shown that the face of a pyramid is always a triangular region.

Problem Set

- The base of a pyramid has area 4. A cross-section that lies in a parallel plane that is distance of 2 from the base plane, has an area of 1. Find the height, h , of the pyramid.

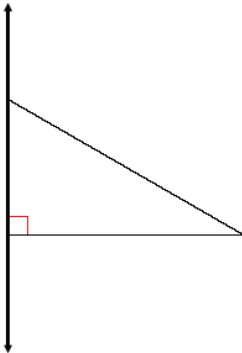


- The base of a pyramid is a trapezoid. The trapezoidal bases have lengths of 3 and 5, and the trapezoid's height is 4. Find the area of the parallel slice that is three-fourths of the way from the vertex to the base.

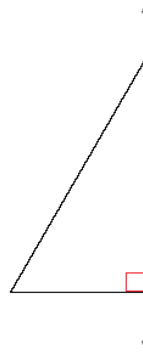


3. A cone has base area 36 cm^2 . A parallel slice 5 cm from the vertex has area 25 cm^2 . Find the height of the cone.
4. Sketch the figures formed if the triangular regions are rotated around the provided axis:

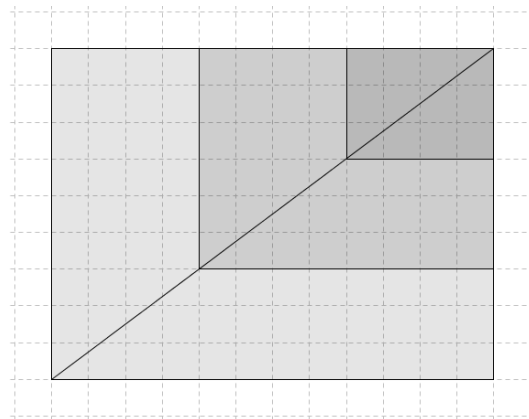
a.



b.

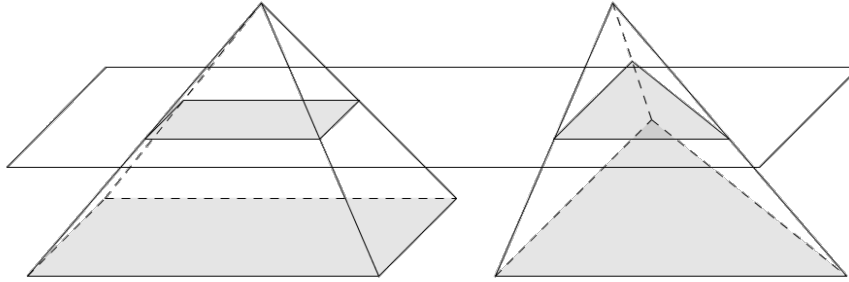


5. Liza drew the top view of a rectangular pyramid with two cross-sections as shown in the diagram and said that her diagram represents one, and only one, rectangular pyramid. Do you agree or disagree with Liza? Explain.



6. A general hexagonal pyramid has height 10 in. A slice 2 in. above the base has area 16 in^2 . Find the area of the base.
7. A general cone has base area 3 units². Find the area of the slice of the cone that is parallel to the base and $\frac{2}{3}$ of the way from the vertex to the base.

8. A rectangular cone and a triangular cone have bases with the same area. Explain why the cross-sections for the cones halfway between the base and the vertex have the same area.



9. The following right triangle is rotated about side AB . What is the resulting figure, and what are its dimensions?

