Lesson 7: General Pyramids and Cones and Their Cross-Sections

Classwork

Opening Exercise

Group the following images by shared properties. What defines each of the groups you have made?

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**Rectangular pyramid:** Given a rectangular region $B$ in a plane $E$ and a point $V$ not in $E$, the *rectangular pyramid with base* $B$ *and vertex* $V$ is the collection of all segments $\overline{VP}$ for any point $P$ in $B.$

**General cone:**  Let $B$ be a region in a plane $E$ and $V$ be a point not in $E$. The *cone with base* $B$ *and vertex* $V$ is the union of all segments $\overline{VP}$ for all points $P$ in $B$ (See Figures 1 and 2).



Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

**Example 1**

In the following triangular pyramid, a plane passes through the pyramid so that it is parallel to the base and results in the cross-section $△A'B'C'$. If the area of $△ABC$ is $25 mm^{2}$, what is the area of $△A'B'C'$?

**Example 2**

In the following triangular pyramid, a plane passes through the pyramid so that it is parallel to the base and results in the cross-section $△A'B'C'$. The altitude from $V$ is drawn; the intersection of the altitude with the base is $X$, and the intersection of the altitude with the cross-section is $X'$. If the distance from $X$ to $V$ is $18 mm$, the distance from $X'$ to $V$ is $12 mm$, and the area of $△A'B'C'$ is $28 mm^{2}$, what is the area of $△ABC$?



**Extension**



Exercise 1

The area of the base of a cone is $16,$ and the height is $10$. Find the area of a cross-section that is distance $5$ from the vertex.

Example 3

**general cone cross-section theorem:** If two general cones have the same base area and the same height, then cross-sections for the general cones the same distance from the vertex have the same area.

State the theorem in your own words.

Use the space below to prove the *general cone cross-section theorem*.

Exercise 2

The following pyramids have equal altitudes, and both bases are equal in area and are coplanar. Both pyramids’ cross-sections are also coplanar. If $BC=3\sqrt{2}$ and $B^{'}C^{'}=2\sqrt{3}$, and the area of $TUVWXYZ$ is $30 units^{2}$, what is the area of cross-section $A'B'C'D'$?

Lesson Summary

**Cone:** Let $B$ be a region in a plane $E$ and $V$ be a point not in $E$. The *cone with base* $B$ *and vertex* $V$ is the union of all segments $\overline{VP}$ for all points $P$ in $B.$

If the base is a polygonal region, then the *cone* is usually called a *pyramid*.

**Rectangular pyramid:** Given a rectangular region $B$ in a plane $E$ and a point $V$ not in $E$, the *rectangular pyramid with base* $B$ *and vertex* $V$ is the union of all segments $\overline{VP}$ for points $P$ in $B.$

**Lateral edge and face of a pyramid:** Suppose the base $B$ of a pyramid with vertex $V$ is a polygonal region and $P\_{i}$ is a vertex of $B$. The segment $\overline{P\_{i}V}$ is called a *lateral edge* of the pyramid. If $\overline{P\_{i}P\_{i+1}}$ is a base edge of the base $B$ (a side of $B$), and $F$ is the union of all segments $\overline{PV}$ for $P$ in $\overline{P\_{i}P\_{i+1}}$ , then $F$ is called a *lateral face* of the pyramid. It can be shown that the face of a pyramid is always a triangular region.

Problem Set

1. The base of a pyramid has area $4$. A cross-section that lies in a parallel plane that is distance of $2$ from the base plane, has an area of $1$. Find the height, $h$, of the pyramid.



1. The base of a pyramid is a trapezoid. The trapezoidal bases have lengths of $3$ and $5$, and the trapezoid’s height is $4$. Find the area of the parallel slice that is three-fourths of the way from the vertex to the base.



1. A cone has base area $36 cm^{2}$. A parallel slice $5 cm$ from the vertex has area $25 cm^{2}$. Find the height of the cone.
2. Sketch the figures formed if the triangular regions are rotated around the provided axis:

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1. Liza drew the top view of a rectangular pyramid with two cross-sections as shown in the diagram and said that her diagram represents one, and only one, rectangular pyramid. Do you agree or disagree with Liza? Explain.



1. A general hexagonal pyramid has height $10 in$. A slice $2 in.$ above the base has area $16 in^{2}$. Find the area of the base.
2. A general cone has base area $3 units^{2}$. Find the area of the slice of the cone that is parallel to the base and $\frac{2}{3}$ of the way from the vertex to the base.
3. A rectangular cone and a triangular cone have bases with the same area. Explain why the cross-sections for the cones halfway between the base and the vertex have the same area.



1. The following right triangle is rotated about side $AB$. What is the resulting figure, and what are its dimensions?