Lesson 6: General Prisms and Cylinders and Their Cross-Sections

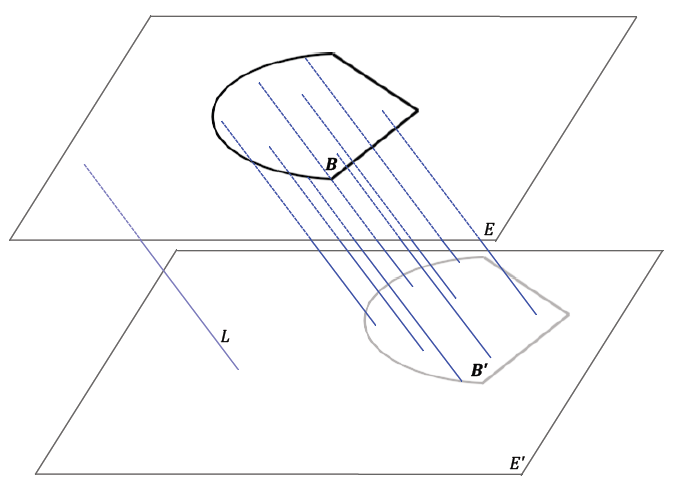
Classwork

Opening Exercise

Sketch a right rectangular prism.

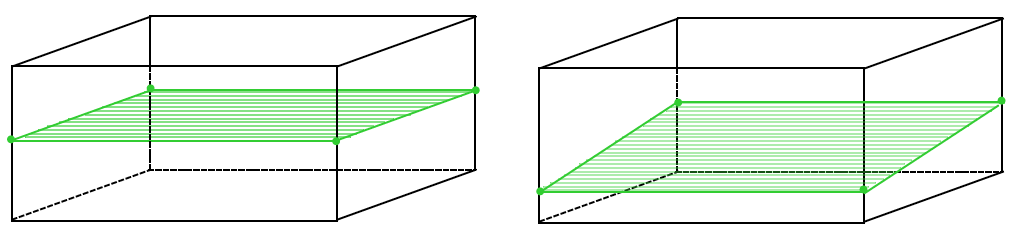
**Right rectangular prism:** Let and be two parallel planes. Let be a rectangular region[[1]](#footnote-1) in the plane . At each point of , consider the segment perpendicular to , joining to a point of the plane . The union of all these segments is called a *right rectangular prism*.

**General cylinder:** (See Figure 1.) Let and be two parallel planes, let be a region[[2]](#footnote-2) in the plane , and let be a line which intersects and but not . At each point of , consider the segment parallel to , joining to a point of the plane . The union of all these segments is called a *general cylinder with base .*



**Figure 1**

Discussion



|  |  |
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| Figure 2 | Figure 3 |
| Example of a cross-section of a prism, where the intersection of a plane with the solid is parallel to the base. | A general intersection of a plane with a prism; sometimes referred to as a slice. |

**Exercise**

Sketch the cross-section for the following figures:

|  |  |  |  |
| --- | --- | --- | --- |
| a. | b. | c. | d. |
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|  |  |  |  |

**Extension**

Figure 4

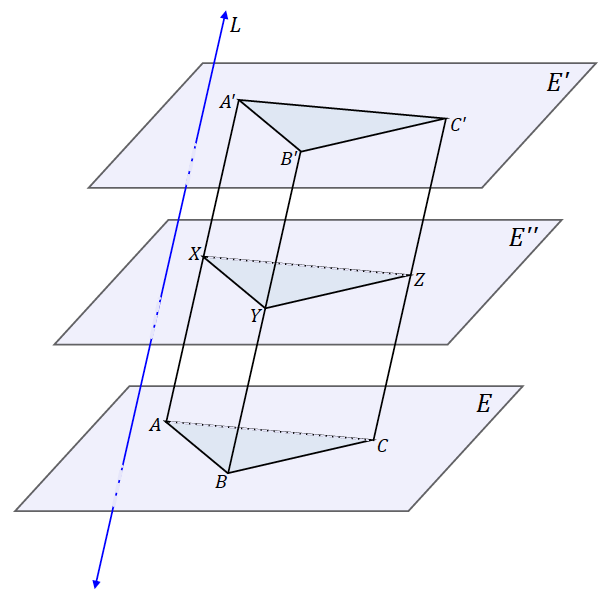
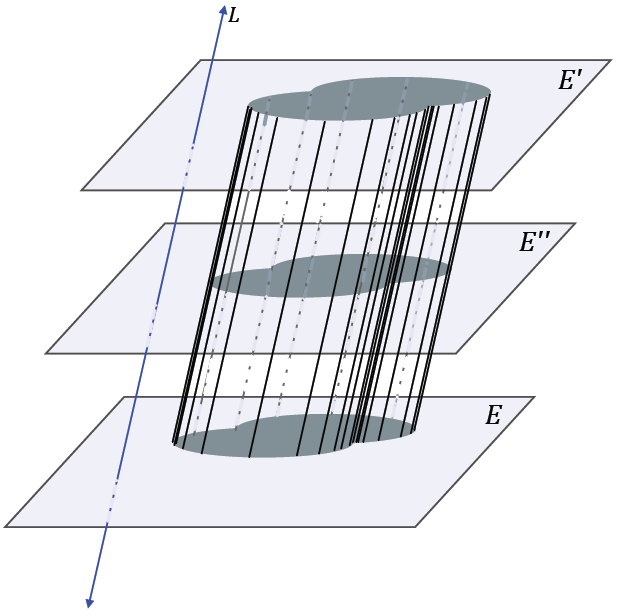


Figure 5

Lesson Summary

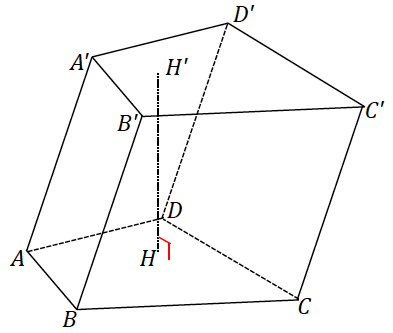
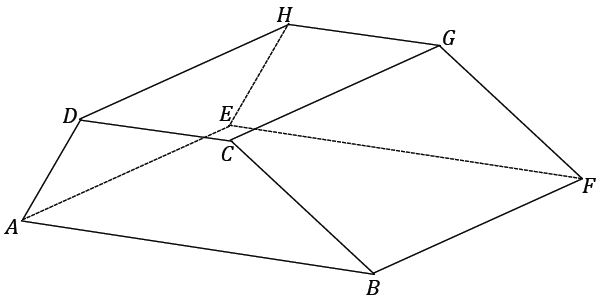
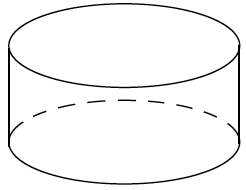
**Relevant Vocabulary**

**Right rectangular prism:** Let and be two parallel planes. Let be a rectangular region in the plane . At each point of , consider the segment perpendicular to , joining to a point of the plane . The union of all these segments is called a *right rectangular prism*.

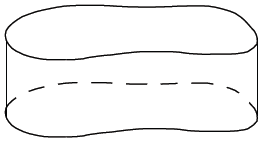
**Lateral edge and face of a prism:**  Suppose the base of a prism is a polygonal region and is a vertex of . Let be the corresponding point in such that is parallel to the line defining the prism. The segment is called a *lateral edge of the prism*. If is a base edge of the base (a side of ), and is the union of all segments parallel to for which is in and is in , then is a *lateral face* *of the prism*. It can be shown that a lateral face of a prism is always a region enclosed by a parallelogram.

**General cylinder:** Let and be two parallel planes, let be a region in the plane , and let be a line which intersects and but not . At each point of , consider the segment parallel to , joining to a point of the plane . The union of all these segments is called a *general cylinder with base .*

Problem Set

1. Complete each statement below by filling in the missing term(s).
   1. The following prism is called a(n) \_\_\_\_\_\_\_\_\_\_\_\_ prism.
   2. If were perpendicular to the plane of the base, then the prism would be called a(n) \_\_\_\_\_\_\_\_\_\_\_\_ prism.
   3. The regions and are called the \_\_\_\_\_\_\_\_\_\_\_\_ of the prism.
   4. is called a(n) \_\_\_\_\_\_\_\_\_\_\_\_.
   5. Parallelogram region is one of four \_\_\_\_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_.
2. The following right prism has trapezoidal base regions; it is a right trapezoidal prism. The lengths of the parallel edges of the base are and , and the nonparallel edges are and ; the height of the trapezoid is . The lateral edge length is . Find the surface area of the prism.
3. The base of the following right cylinder has a circumference of and a lateral edge of . What is the radius of the base? What is the lateral area of the right cylinder?

1. The following right general cylinder has a lateral edge of length , and the perimeter of its base is . What is the lateral area of the right general cylinder?

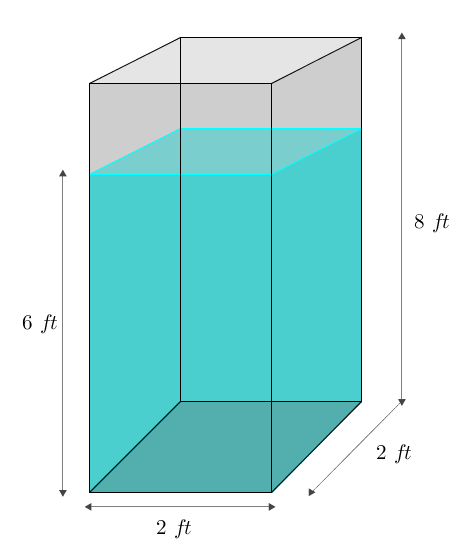


1. A right prism has base area and volume . Find the prism’s height, .
2. Sketch the figures formed if the rectangular regions are rotated around the provided axis.

|  |  |
| --- | --- |
|  |  |

1. A cross-section is taken parallel to the bases of a general cylinder and has an area of . If the height of the cylinder is , what is the volume of the cylinder? Explain your reasoning.
2. A general cylinder has a volume of . What is one possible set of dimensions of the base and height of the cylinder if all cross-sections parallel to its bases are …
   1. Rectangles?
   2. Right triangles?
   3. Circles?
3. A general hexagonal prism is given. If is a plane that is parallel to the planes containing the base faces of the prism, how does meet the prism?
4. Two right prisms have similar bases. The first prism has height and volume . A side on the base of the first prism has length and the corresponding side on the base of the second prism has length . If the height of the second prism is , what is its volume?

1. A tank is the shape of a right rectangular prism with base and height . The tank is filled with water to a depth of . A person of height jumps in and stands on the bottom. About how many inches will the water be over the person’s head? Make reasonable assumptions.



1. (Fill in the blank.) A rectangular region is the union of a rectangle and \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ . [↑](#footnote-ref-1)
2. In Grade 8, a *region* refers to a *polygonal region* (triangle, quadrilateral, pentagon, and hexagon) or a *circular region,* or regions that can be decomposed into such regions. [↑](#footnote-ref-2)