Topic B:

**Volume**

G-GMD.A.1, G-GMD.A.3, G-GMD.B.4, G-MG.A.1, G-MG.A.2, G-MG.A.3

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| Focus Standards: | G-GMD.A.1 | Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri’s principle, and informal limit arguments.* |
|  | G-GMD.A.3 | Use volume formulas for cylinders, pyramids, cones and spheres to solve problems.★ |
|  | G-GMD.B.4 | Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. |
|  | G-MG.A.1 | Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★ |
|  | G-MG.A.2 | Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★ |
|  | G-MG.A.3 | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★ |
| Instructional Days: | 9 |  |
| Lesson 5: | Three-Dimensional Space (E)[[1]](#footnote-1) |
| Lesson 6:  | General Prisms and Cylinders and Their Cross-Sections (E) |
| Lesson 7: | General Pyramids and Cones and Their Cross-Sections (S) |
| Lesson 8: | Definition and Properties of Volume (S) |
| Lesson 9: | Scaling Principle for Volumes (P) |
| Lesson 10:  | The Volume of Prisms and Cylinders and Cavalieri’s Principle (S) |
| Lesson 11: | The Volume Formula of a Pyramid and Cone (E) |
| Lesson 12: | The Volume Formula of a Sphere (S) |
| Lesson 13: | How Do 3D Printers Work? (S) |

With a reference to area established in Topic A, students study volume in Topic B. In Grade 8, volume is treated independent of the subtle problems that arise when we attempt to measure the volume of figures other than rectangular solids. From an advanced mathematical perspective, area and volume are conceptually very close in that Jordan measure provides a good foundation, but there are profound differences between area and volume that show up mathematically only when we consider the problem of cutting bodies along planes and reassembling them. Two bodies of the same volume might not be
“equi-decomposable” in this sense. This, of course, is much more advanced an idea than anything in the curriculum, but it is one of the mathematical reasons Cavalieri's principle is indispensable. In contrasting Grade 8 with Module 3, the role of this principle is a prominent difference. More generally, understanding and predicting the shapes of cross-sections of three-dimensional figures—though it was done in Grade 7—is a complex skill that needs a lot of work to fully develop. We return to that with a level of sophistication that was absent in Grade 7.

In Lesson 5, students study the basic properties of two-dimensional and three-dimensional space, noting how ideas shift between the dimensions. For example, in two-dimensional space, two lines perpendicular to the same line are parallel, but in three-dimensional space we consider how two *planes* perpendicular to the same line are parallel. In Lesson 6, students learn that general cylinders are the parent category for prisms, circular cylinders, right cylinders, and oblique cylinders (MP.6). Students also study why the cross-section of a cylinder is congruent to its base (**G-GMD.B.4**). In Lesson 7, students study the explicit definition of a cone and learn what distinguishes pyramids from general cones. Students also see how dilations explain why a cross-section taken parallel to the base of a cone is similar to the base (**G-GMD.B.4**, MP.7). Lesson 8 demonstrates the properties of volume, which are analogous to the properties of area (seen in Lesson 2). Students reason why the volume of any right triangular prism has the same volume formula as that of a right triangular prism with a right triangle as a base. This leads to the generalization of the volume formula for any right cylinder (**G-GMD.A.1**, **G-GMD.A.3**). In Lesson 9, students examine the scaling principle for volume (they have seen the parallel situation regarding area in Lesson 3) and see that a solid scaled by factors $a$, $b$, and $c$ in three perpendicular directions will result in a volume multiplied by a factor of $abc$. In Lesson 10, students learn Cavalieri’s principle, which describes the relationship between cross-sections of two solids and their respective volumes. If two solids are included between two parallel planes, and cross-sections taken parallel to the bases are of equal area at every level, then the volumes of the solids must be equal. Cavalieri’s principle is used to reason why the volume formula of any cylinder is area of $base×height $(**G-GMD.A.1**). Lesson 11 focuses on the derivation of the volume formulas for cones, and Lesson 12 focuses on the derivation of the volume formula for spheres, which depends partly on the volume formula of a cone
(**G-GMD.A.1**). Lesson 13 is a look at 3D printers and ultimately how the technology is linked to Cavalieri’s principle.

Module 3 is a natural place to see geometric concepts in modeling situations. Modeling-based problems are found throughout Topic B and include the modeling of real-world objects, the application of density, the occurrence of physical constraints, and issues regarding cost and profit (**G-MG.A.1**, **G-MG.A.2**, **G-MG.A.3**).

1. Lesson Structure Key: **P**-Problem Set Lesson, **M**-Modeling Cycle Lesson, **E-**Exploration Lesson, **S-**Socratic Lesson [↑](#footnote-ref-1)