



Topic A:

Area

G-GMD.A.1

Focus Standard:	G-GMD.A.1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. <i>Use dissection arguments, Cavalieri's principle, and informal limit arguments.</i>
Instructional Days:	4	
Lesson 1:	What Is Area? (E) ¹	
Lesson 2:	Properties of Area (E)	
Lesson 3:	The Scaling Principle for Area (E)	
Lesson 4:	Proving the Area of a Disk (S)	

The topic of area must be revisited in order to have a conversation about figures in three dimensions; we first have the necessary discussion around area. Area is introduced in Grade 3, but only figures that are easy to “fill” with units are considered. In Grade 5, the need to use parts of unit squares for some figures is understood, and is applied in Grade 6. In Grade 7, students realize that a figure may have an area even if, like the disk, it cannot be decomposed into a finite number of unit squares, but (appropriately) this is treated at the intuitive level. As the grades progress, the link between area as a measurable geometric quantity and area represented by numbers for calculations is elaborated. The culmination is a universal method for measuring areas, even when they are not finite unions of unit squares or simple parts of unit squares. Mathematicians refer to this as Jordan measure. While this may seem like an intimidating idea to introduce at this level, it can be thought of simply as the well-known properties of area that students are already familiar with. This is not only an intuitively acceptable approach to area, but it is completely rigorous at the university level. The concept forms an important bridge to calculus, as Jordan measure is the idea employed in defining the Riemann integral.

In this topic, Lesson 1 shows how finding the area of a curved figure can be approximated by rectangles and triangles. By refining the size of the rectangles and triangles, the approximation of the area becomes closer to the actual area. Students experience a similar process of approximation in Grade 8 (Module 7, Lesson 14) in order to estimate π . The informal limit argument prepares students for the development of volume

¹ Lesson Structure Key: **P**-Problem Set Lesson, **M**-Modeling Cycle Lesson, **E**-Exploration Lesson, **S**-Socratic Lesson

formulas for cylinders and cones and foreshadows ideas that students will formally explore in calculus. This process of approximation is important to developing the volume formula of cylinders and cones. In Lesson 2, students study the basic properties of area using set notation; in Topic B they will see how the properties are analogous to those of volume. In Lesson 3, students study the scaling principle, which states that if a planar region is scaled by factors a and b in two perpendicular directions, then its area is multiplied by a factor of $a \times b$. Again, we study this in two dimensions to set the stage for three dimensions when we scale solids. Finally, in Lesson 4, students develop the formula for the area of a disk, and just as in Lesson 1, incorporate an approximation process. Students approximate the area of the disk, or circle, by inscribing a polygon within the circle, and consider how the area of the polygonal region changes as the number of sides increases and the polygon looks more and more like the disk it is inscribed within.