Lesson 22: Multiplying and Dividing Expressions with Radicals

Student Outcomes

- Students multiply and divide expressions that contain radicals to simplify their answers.
- Students rationalize the denominator of a number expressed as a fraction.

Lesson Notes

Exercises 1–5 and Discussion are meant to remind students of what they learned about roots in Grade 8 and Algebra I. In Grade 8, students learned the notation related to square roots and understood that the square root symbol automatically denotes the positive root (Grade 8, Module 7). In Algebra I, students used both the positive and negative roots of a number to find the location of the roots of a quadratic function. In this lesson, we will review what we learned about roots in Grade 8, Module 7, Lesson 4, because of the upcoming work with special triangles in this module. For example, we want students to be clear that $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ when they are writing the trigonometric ratios of right triangles. To achieve this understanding, students must learn how to rationalize the denominator of numbers expressed as fractions. It is also important for students to get a sense of the value of a number. When a radical is in the denominator or is not simplified, it is more challenging to estimate its value, e.g., $\sqrt{3750}$ compared to $25\sqrt{6}$.

For students who are struggling with the concepts of multiplying and dividing expressions with radicals, it may be necessary to divide the lesson so that multiplication is the focus one day and division the next. This lesson is a stepping stone, as it moves students toward an understanding of how to rewrite expressions involving radical and rational exponents using the properties of exponents (**N.RN.A.2**), which will not be mastered until Algebra II.

The lesson focuses on simplifying expressions and solving equations that contain terms with roots. By the end of the lesson, students should understand that one reason we rationalize the denominator of a number expressed as a fraction is to better estimate the value of the number. For example, one can more accurately estimate the value of $\frac{3}{\sqrt{3}}$ when written as $\frac{3\sqrt{3}}{3}$. Further, putting numbers in this form allows us to more easily recognize when numbers can be combined. For example, if you had to add $\sqrt{3}$ and $\frac{1}{\sqrt{3}}$, you may not recognize that they can be combined until $\frac{1}{\sqrt{3}}$ is rewritten as $\frac{\sqrt{3}}{3}$. Then, the sum of $\sqrt{3}$ and $\frac{\sqrt{3}}{3}$ is $\frac{4\sqrt{3}}{3}$. As a teacher, it is easier to check answers when there is an expected standard form such as a rationalized expression.

Classwork

Exercises 1–5 (8 minutes)

The first three exercises review square roots that are perfect squares. The last two exercises require students to compare the value of two radical expressions and make a conjecture about their relationship. These last two exercises exemplify what will be studied in this lesson. Students may need to be reminded that the square root symbol automatically denotes the positive root of the number.

Scaffolding:

- Some students may need to review the perfect squares. A reproducible sheet for squares of numbers 1–30 is provided at the end of the lesson.
- Consider doing a fluency activity that allows students to learn their perfect squares up to 30. This may include choral recitation.
- English language learners may benefit from choral practice with the word *radical*.

Exercises 1–5

Simplify as much as possible.

1. $\sqrt{17^2} =$

   $\sqrt{17^2} = 17$

2. $\sqrt{5^{10}} =$

   $$\begin{aligned}\sqrt{5^{10}} &= \sqrt{5^2} \times \sqrt{5^2} \times \sqrt{5^2} \times \sqrt{5^2} \times \sqrt{5^2}\\ &= 5 \times 5 \times 5 \times 5 \times 5\\ &= 5^5\end{aligned}$$

3. $\sqrt{4x^4} =$

   $$\begin{aligned}\sqrt{4x^4} &= \sqrt{4} \times \sqrt{x^2} \times \sqrt{x^2}\\ &= 2 \times x \times x\\ &= 2|x|^2\end{aligned}$$

4. **Complete parts (a) through (c).**

   a. **Compare the value of $\sqrt{36}$ to the value of $\sqrt{9} \times \sqrt{4}$.**

   The value of the two expressions is equal. The square root of 36 is 6, and the product of the square roots of 9 and 4 is also 6.

   b. **Make a conjecture about the validity of the following statement. For nonnegative real numbers a and b, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$. Explain.**

   Answers will vary. Students should say that the statement $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ is valid because of the problem that was just completed: $\sqrt{36} = \sqrt{9} \times \sqrt{4} = 6$.

   c. **Does your conjecture hold true for $a = -4$ and $b = -9$?**

   No, the conjecture is not true when the numbers are negative because we cannot take the square root of a negative number. $\sqrt{(-4)(-9)} = \sqrt{36} = 6$, but we cannot calculate $\sqrt{-4} \times \sqrt{-9}$ in order to compare.

5. **Complete parts (a) through (c).**

   a. **Compare the value of $\sqrt{\frac{100}{25}}$ to the value of $\frac{\sqrt{100}}{\sqrt{25}}$.**

   The value of the two expressions is equal. The fraction $\frac{100}{25}$ simplifies to 4, and the square root of 4 is 2. The square root of 100 divided by the square root of 25 is equal to $\frac{10}{5}$, which is equal to 2.

b. **Make a conjecture about the validity of the following statement. For nonnegative real numbers a and b, when $b \neq 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$. Explain.**

Answers will vary. Students should say that the statement $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ is valid because of the problem that was just completed: $\sqrt{\frac{100}{25}} = \frac{\sqrt{100}}{\sqrt{25}} = 2$.

c. **Does your conjecture hold true for $a = -100$ and $b = -25$?**

No, the conjecture is not true when the numbers are negative because we cannot take the square root of a negative number. $\sqrt{\frac{-100}{-25}} = \sqrt{4} = 2$, but we cannot calculate $\frac{\sqrt{-100}}{\sqrt{-25}}$ in order to compare.

Discussion (8 minutes)

Debrief Exercises 1–5 by reminding students of the definition for square root, the facts given by the definition, and the rules associated with square roots for positive radicands. Whenever possible, elicit the facts and definitions from students based on their work in Exercises 1–5. Within this discussion is the important distinction between *a* square root of a number and *the* square root of a number. *A* square root of a number may be negative; however, *the* square root of a number always refers to the principle square root or the positive root of the number.

- Definition of *the* square root: If $x \geq 0$, then $\sqrt{x}$ is the nonnegative number p so that $p^2 = x$. This definition gives us four facts. The definition should not be confused with finding *a* square root of a number. For example -2 is *a* square root of 4, but *the* square root of 4, i.e., $\sqrt{4}$, is 2.

Consider asking students to give an example of each fact using concrete numbers. Sample responses are included below each fact.

Fact 1: $\sqrt{a^2} = a$ if $a \geq 0$

- $\sqrt{12^2} = 12$*, for any positive squared number in the radicand.*

Fact 2: $\sqrt{a^2} = -a$ if $a < 0$

This may require additional explanation because students will see the answer as "negative a," as opposed to the opposite of a. For this fact, we assume that a is a negative number; therefore, $-a$ is a positive number. It is similar to how we think about the absolute value of a number a: $|a| = a$ if $a > 0$, but $|a| = -a$ if $a < 0$. Simply put, the minus sign is just telling us we need to take the opposite of the negative number a to produce the desired result, i.e., $-(-a) = a$.

- $\sqrt{(-5)^2} = 5$*, for any negative squared number in the radicand.*

Fact 3: $\sqrt{a^2} = |a|$ for all real numbers a.

- $\sqrt{13^2} = |13|$*, and* $\sqrt{(-13)^2} = |-13|$.

Fact 4: $\sqrt{a^{2n}} = \sqrt{(a^n)^2} = a^n$ when a^n is nonnegative.

- $\sqrt{7^{16}} = \sqrt{(7^8)^2} = 7^8$

Consider asking students which of the first five exercises used Rule 1.

- When $a \geq 0, b \geq 0$, and $c \geq 0$, then the following rules can be applied to square roots.

  Rule 1: $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$. A consequence of rule 1 and the associative property gives us the following: $\sqrt{abc} = \sqrt{a(bc)} = \sqrt{a} \cdot \sqrt{bc} = \sqrt{a} \cdot \sqrt{b} \cdot \sqrt{c}$.

  Rule 2: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ when $b \neq 0$.

- We want to show that $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ for $a \geq 0$ and $b \geq 0$. To do so, we will use the definition of square root.

Consider allowing time for students to discuss with partners how they can prove rule 1. Shown below are three proofs of Rule 1. You can choose to share one or all with the class.

The proof of rule 1: Let p be the nonnegative number so that $p^2 = a$, and let q be the nonnegative number so that $q^2 = b$. Then, $\sqrt{ab} = pq$ because pq is nonnegative (it is the product of two nonnegative numbers), and $(pq)^2 = pqpq = p^2q^2 = ab$. Then by definition, $\sqrt{ab} = pq = \sqrt{a} \cdot \sqrt{b}$. Since both sides equal pq, the equation is true.

The proof of rule 1: Let $C = \sqrt{a} \cdot \sqrt{b}$, and let $D = \sqrt{ab}$. We need to show that $C = D$. Given positive numbers C, D and exponent 2, if we can show that $C^2 = D^2$, then we know that $C = D$, and rule 1 will be proved.

Consider asking students why it is true that if we can show that $C^2 = D^2$, then we know that $C = D$. Students should refer to what they know about the definition of exponents. That is, since $C^2 = C \times C$ and $D^2 = D \times D$ and $C \times C = D \times D$, then C must be the same number as D.

- With that goal in mind, we take each of $C = \sqrt{a}\sqrt{b}$ and $D = \sqrt{ab}$, and by the multiplication property of equality, we raise both sides of each equation to a power of 2.

$$\begin{aligned} C^2 &= \left(\sqrt{a} \cdot \sqrt{b}\right)^2 \\ &= \left(\sqrt{a} \cdot \sqrt{b}\right) \cdot \left(\sqrt{a} \cdot \sqrt{b}\right) \\ &= \sqrt{a} \cdot \sqrt{a} \cdot \sqrt{b} \cdot \sqrt{b} \\ &= ab \end{aligned}$$

$$\begin{aligned} D^2 &= \left(\sqrt{ab}\right)^2 \\ &= \sqrt{ab} \cdot \sqrt{ab} \\ &= ab \end{aligned}$$

Since $C^2 = D^2$ implies $C = D$, then $\sqrt{ab} = \sqrt{a}\sqrt{b}$.

The proof of rule 1: Let $C, D > 0$. If $C^2 = D^2$, then $C = D$. Assume $C^2 = D^2$, then $C^2 - D^2 = 0$. By factoring the difference of squares, we have $(C + D)(C - D) = 0$. Since both C and D are positive, then $C + D > 0$, which means that $C - D$ must be equal to zero because of the zero product property. Since $C - D = 0$, then $C = D$.

Example 1 (4 minutes)

- We can use rule 1 to rationalize the denominators of fractional expressions. One reason we do this is so that we can better estimate the value of a number. For example, if we know that $\sqrt{2} \approx 1.414$, what is the value of $\frac{1}{\sqrt{2}}$? Isn't it is easier to determine the value of $\frac{\sqrt{2}}{2}$? The fractional expressions $\frac{1}{\sqrt{2}}$ and $\frac{\sqrt{2}}{2}$ are equivalent. Notice that the first expression has the irrational number $\sqrt{2}$ as its denominator, and the second expression has the rational number 2 as its denominator. What we will learn today is how to rationalize the denominator of a fractional expression using rule 1.
- Another reason to rationalize the denominators of fractional expressions is because putting numbers in this form allows us to more easily recognize when numbers can be combined. For example, if you have to add $\sqrt{3}$ and $\frac{1}{\sqrt{3}}$, you may not recognize that they can be combined until $\frac{1}{\sqrt{3}}$ is rewritten as $\frac{\sqrt{3}}{3}$. Then, the sum of $\sqrt{3}$ and $\frac{\sqrt{3}}{3}$ is $\frac{4\sqrt{3}}{3}$.
- We want to express numbers in their *simplest radical form*. An expression is in its *simplest radical form* when the radicand (the expression under the radical sign) has no factor that can be raised to a power greater than or equal to the index (either 2 or 3), and there is no radical in the denominator.
- Using rule 1 for square roots, we can simplify expressions that contain square roots by writing the factors of the number under the square root sign as products of perfect squares, if possible. For example, to simplify $\sqrt{75}$, we consider all of the factors of 75, focusing on those factors that are perfect squares. Which factors should we use?
  - *We should use* 25 *and* 3 *because* 25 *is a perfect square.*

Then,

$$\begin{aligned}\sqrt{75} &= \sqrt{25} \cdot \sqrt{3} \\ &= 5\sqrt{3}.\end{aligned}$$

> *Scaffolding:*
>
> Consider showing multiple simple examples. For example:
>
> $$\begin{aligned}\sqrt{28} &= \sqrt{4} \cdot \sqrt{7} \\ &= 2\sqrt{7} \\ \sqrt{45} &= \sqrt{9} \cdot \sqrt{5} \\ &= 3\sqrt{5} \\ \sqrt{32} &= \sqrt{16} \cdot \sqrt{2} \\ &= 4\sqrt{2}\end{aligned}$$

Example 2 (2 minutes)

In Example 2, we first use rule 2 to rewrite a number as a rational expression, then use rule 1 to rationalize a denominator, that is, rewrite the denominator as an integer. We have not yet proved this rule because it is an exercise in the problem set. Consider mentioning this fact to students.

- Rules 1 and 2 for square roots are used to rationalize denominators of fractional expressions.

You may want to ask students what it means to "rationalize the denominator." Students should understand that "rationalizing the denominator" means expressing it as an integer.

- Consider the expression $\sqrt{\frac{3}{5}}$. By rule 2, $\sqrt{\frac{3}{5}} = \frac{\sqrt{3}}{\sqrt{5}}$. We want to write an expression that is equivalent to $\frac{\sqrt{3}}{\sqrt{5}}$ with a rational number for the denominator.

$$\frac{\sqrt{3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{3}\sqrt{5}}{\sqrt{5}\sqrt{5}}$$ By multiplication rule for fractional expressions

$$= \frac{\sqrt{15}}{\sqrt{25}}$$ By rule 1

$$= \frac{\sqrt{15}}{5}.$$

Example 3 (3 minutes)

MP.3

- Demarcus found the scale factor of a dilation to be $\frac{1}{\sqrt{2}}$. When he compared his answer to Yesenia's, which was $\frac{\sqrt{2}}{2}$, he told her that one of them must have made a mistake. Show work and provide an explanation to Demarcus and Yesenia that proves they are both correct.
  - *Student work:*

$$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{1\sqrt{2}}{\sqrt{2}\sqrt{2}}$$ By multiplication rule for fractional expressions

$$= \frac{\sqrt{2}}{\sqrt{4}}$$ By rule 1

$$= \frac{\sqrt{2}}{2}$$ By definition of *square root*

If Demarcus were to rationalize the denominator of his answer, he would see that it is equal to Yesenia's answer. Therefore, they are both correct.

Example 4 (5 minutes)

- Assume $x > 0$. Rationalize the denominator of $\frac{x}{\sqrt{x^3}}$, and then simplify your answer as much as possible.

Provide time for students to work independently or in pairs. Use the question below if necessary. Seek out students who multiplied by different factors to produce an equivalent fractional expression to simplify this problem. For example, some students may have multiplied by $\frac{\sqrt{x^3}}{\sqrt{x^3}}$, while others may have used $\frac{\sqrt{x}}{\sqrt{x}}$ or some other fractional expression that would produce an exponent of x with an even number which can be simplified. Have students share their work and compare their answers.

- We need to multiply $\sqrt{x^3}$ by a number so that it becomes a perfect square. What should we multiply by?

Students may say to multiply by $\sqrt{x^3}$ because that is what was done in the two previous examples. If so, finish the problem that way, and then show that we can multiply by $\sqrt{x}$ and get the same answer. Ask students why both methods work. They should mention equivalent expressions and the role that the number $\frac{\sqrt{x^3}}{\sqrt{x^3}}$ or $\frac{\sqrt{x}}{\sqrt{x}}$ plays in producing the equivalent expression.

- *Student work:*

$$\frac{x}{\sqrt{x^3}} \times \frac{\sqrt{x^3}}{\sqrt{x^3}} = \frac{x\sqrt{x^3}}{\sqrt{x^3}\sqrt{x^3}} = \frac{x\sqrt{x^3}}{\sqrt{x^6}} = \frac{xx\sqrt{x}}{x^3} = \frac{x^2\sqrt{x}}{x^3} = \frac{\sqrt{x}}{x}$$

$$\frac{x}{\sqrt{x^3}} \times \frac{\sqrt{x}}{\sqrt{x}} = \frac{x\sqrt{x}}{\sqrt{x^3}\sqrt{x}} = \frac{x\sqrt{x}}{\sqrt{x^4}} = \frac{x\sqrt{x}}{x^2} = \frac{\sqrt{x}}{x}$$

Exercises 6–17 (7 minutes)

You can choose to have students work through all of the exercises in this set or select problems for students to complete based on their level. Students who are struggling should complete Exercises 6–10. Students who are on level should complete Exercises 9–13. Students who are accelerated should complete Exercises 13–16. All students should attempt to complete Exercise 17.

Exercises 6–17

Simplify each expression as much as possible, and rationalize denominators when applicable.

6. $\sqrt{72} =$

$$\sqrt{72} = \sqrt{36} \cdot \sqrt{2} = 6\sqrt{2}$$

7. $\sqrt{\frac{17}{25}} =$

$$\sqrt{\frac{17}{25}} = \frac{\sqrt{17}}{\sqrt{25}} = \frac{\sqrt{17}}{5}$$

8. $\sqrt{32x} =$

$$\sqrt{32x} = \sqrt{16}\sqrt{2x} = 4\sqrt{2x}$$

9. $\sqrt{\frac{1}{3}} =$

$$\sqrt{\frac{1}{3}} = \frac{\sqrt{1}}{\sqrt{3}} = \frac{\sqrt{1}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{9}} = \frac{\sqrt{3}}{3}$$

10. $\sqrt{54x^2} =$

$$\sqrt{54x^2} = \sqrt{9}\sqrt{6}\sqrt{x^2}$$
$$= 3x\sqrt{6}$$

11. $\dfrac{\sqrt{36}}{\sqrt{18}} =$

$$\frac{\sqrt{36}}{\sqrt{18}} = \sqrt{\frac{36}{18}}$$
$$= \sqrt{2}$$

12. $\sqrt{\dfrac{4}{x^4}} =$

$$\sqrt{\frac{4}{x^4}} = \frac{\sqrt{4}}{\sqrt{x^4}}$$
$$= \frac{2}{x^2}$$

13. $\dfrac{4x}{\sqrt{64x^2}} =$

$$\frac{4x}{\sqrt{64x^2}} = \frac{4x}{8x}$$
$$= \frac{1}{2}$$

14. $\dfrac{5}{\sqrt{x^7}} =$

$$\frac{5}{\sqrt{x^7}} = \frac{5}{\sqrt{x^7}} \times \frac{\sqrt{x}}{\sqrt{x}}$$
$$= \frac{5\sqrt{x}}{\sqrt{x^8}}$$
$$= \frac{5\sqrt{x}}{x^4}$$

15. $\sqrt{\dfrac{x^5}{2}} =$

$$\sqrt{\frac{x^5}{2}} = \frac{\sqrt{x^5}}{\sqrt{2}}$$
$$= \frac{x^2\sqrt{x}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{x^2\sqrt{2x}}{2}$$

16. $\dfrac{\sqrt{18x}}{3\sqrt{x^5}} =$

$$\frac{\sqrt{18x}}{3\sqrt{x^5}} = \frac{\sqrt{9}\sqrt{2x}}{3x^2\sqrt{x}}$$
$$= \frac{3\sqrt{2x}}{3x^2\sqrt{x}}$$
$$= \frac{\sqrt{2x}}{x^2\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{x}}$$
$$= \frac{x\sqrt{2}}{x^3}$$
$$= \frac{\sqrt{2}}{x^2}$$

17. The captain of a ship recorded the ship's coordinates, then sailed north and then west, and then recorded the new coordinates. The coordinates were used to calculate the distance they traveled, $\sqrt{578}$ km. When the captain asked how far they traveled, the navigator said, "About 24 km." Is the navigator correct? Under what conditions might he need to be more precise in his answer?

Sample student responses:

The number $\sqrt{578}$ is close to the perfect square $\sqrt{576}$. The perfect square $\sqrt{576} = 24$; therefore, the navigator is correct in his estimate of distance traveled.

When the number $\sqrt{578}$ is simplified, the result is $17\sqrt{2}$. The number 578 has factors of 289 and 2, then:

$$\begin{aligned}\sqrt{578} &= \sqrt{289}\sqrt{2}\\ &= 17\sqrt{2}\\ &= 24.04163\ldots\\ &\approx 24.\end{aligned}$$

Yes, the navigator is correct in his estimate of distance traveled.

A more precise answer may be needed if the captain were looking for a particular location, such as the location of a shipwreck or buried treasure.

Closing (4 minutes)

Ask the following questions. You may choose to have students respond in writing, to a partner, or to the whole class.

- What are some of the basic facts and rules related to square roots?
  - *The basic facts about square roots show us how to simplify a square root when it is a perfect square. For example, $\sqrt{5^2} = 5$, $\sqrt{(-5)^2} = 5$, $\sqrt{(-5)^2} = |-5|$, and $\sqrt{5^{12}} = \sqrt{(5^6)^2} = 5^6$. The rules allow us to simplify square roots. Rule 1 shows that we can rewrite radicands as factors and simplify the factors, if possible. Rule 2 shows us that the square root of a fractional expression can be expressed as the square root of the numerator divided by the square root of a denominator.*
- What does it mean to rationalize the denominator of a fractional expression? Why might we want to do it?
  - *Rationalizing a denominator means that the fractional expression must be expressed with a rational number in the denominator. We might want to rationalize the denominator of a fractional expression to better estimate the value of the number. Another reason is to verify whether two numbers are equal or can be combined.*

Exit Ticket (4 minutes)

Name ______________________________ Date ______________

Lesson 22: Multiplying and Dividing Expressions with Radicals

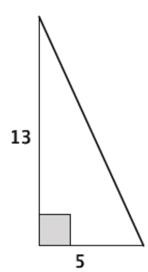
Exit Ticket

Write each expression in its simplest radical form.

1. $\sqrt{243} =$

2. $\sqrt{\frac{7}{5}} =$

3. Teja missed class today. Explain to her how to write the length of the hypotenuse in simplest radical form.



Exit Ticket Sample Solutions

Write each expression in its simplest radical form.

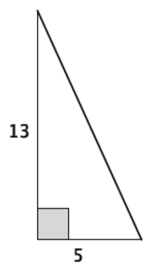
1. $\sqrt{243} =$

$$\sqrt{243} = \sqrt{81} \cdot \sqrt{3} = 9\sqrt{3}$$

2. $\sqrt{\frac{7}{5}} =$

$$\sqrt{\frac{7}{5}} = \frac{\sqrt{7}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{7}\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{\sqrt{35}}{\sqrt{25}} = \frac{\sqrt{35}}{5}$$

3. **Teja missed class today. Explain to her how to write the length of the hypotenuse in simplest radical form.**



Use the Pythagorean Theorem to determine the length of the hypotenuse, c:

$$5^2 + 13^2 = c^2$$
$$25 + 169 = c^2$$
$$194 = c^2$$
$$\sqrt{194} = c$$

To simplify the square root, rewrite the radicand as a product of its factors. The goal is to find a factor that is a perfect square and can then be simplified. There are no perfect square factors of the radicand; therefore, the length of the hypotenuse in simplest radical form is $\sqrt{194}$.

Problem Set Sample Solutions

Express each number in its simplest radical form.

1. $\sqrt{6} \cdot \sqrt{60} =$

$$\sqrt{6} \cdot \sqrt{60} = \sqrt{6} \cdot \sqrt{6} \cdot \sqrt{10} = 6\sqrt{10}$$

2. $\sqrt{108} =$

$$\sqrt{108} = \sqrt{9} \cdot \sqrt{4} \cdot \sqrt{3} = 3 \cdot 2\sqrt{3} = 6\sqrt{3}$$

3. **Pablo found the length of the hypotenuse of a right triangle to be $\sqrt{45}$. Can the length be simplified? Explain.**

$$\sqrt{45} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$$

Yes, the length can be simplified because the number 45 has a factor that is a perfect square.

4. $\sqrt{12x^4} =$

$$\sqrt{12x^4} = \sqrt{4}\sqrt{3}\sqrt{x^4}$$
$$= 2x^2\sqrt{3}$$

5. Sarahi found the distance between two points on a coordinate plane to be $\sqrt{74}$. Can this answer be simplified? Explain.

The number 74 can be factored, but none of the factors are perfect squares, which are necessary to simplify. Therefore, $\sqrt{74}$ cannot be simplified.

6. $\sqrt{16x^3} =$

$$\sqrt{16x^3} = \sqrt{16}\sqrt{x^2}\sqrt{x}$$
$$= 4|x|\sqrt{x}$$

7. $\frac{\sqrt{27}}{\sqrt{3}} =$

$$\frac{\sqrt{27}}{\sqrt{3}} = \sqrt{\frac{27}{3}}$$
$$= \sqrt{9}$$
$$= 3$$

8. Nazem and Joffrey are arguing about who got the right answer. Nazem says the answer is $\frac{1}{\sqrt{3}}$, and Joffrey says the answer is $\frac{\sqrt{3}}{3}$. Show and explain that their answers are equivalent.

$$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$= \frac{\sqrt{3}}{3}$$

If Nazem were to rationalize the denominator in his answer, he would see that it is equal to Joffrey's answer.

9. $\sqrt{\frac{5}{8}} =$

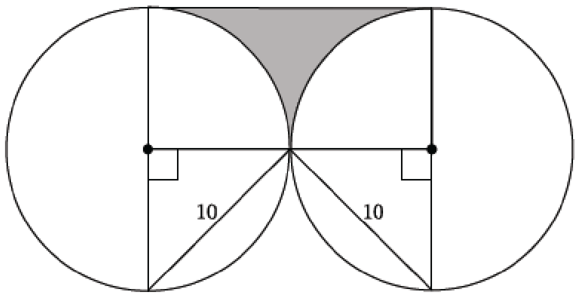
$$\sqrt{\frac{5}{8}} = \frac{\sqrt{5}}{\sqrt{8}} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{\sqrt{10}}{\sqrt{16}}$$
$$= \frac{\sqrt{10}}{4}$$

10. **Determine the area of a square with side length $2\sqrt{7}$ in.**

$$\begin{aligned} A &= \left(2\sqrt{7}\right)^2 \\ &= 2^2\left(\sqrt{7}\right)^2 \\ &= 4(7) \\ &= 28 \end{aligned}$$

The area of the square is 28 in^2.

11. **Determine the exact area of the shaded region shown below.**



Let r be the length of the radius.

By special triangles or the Pythagorean theorem, $r = 5\sqrt{2}$.

The area of the rectangle containing the shaded region is

$$A = 2\left(5\sqrt{2}\right)\left(5\sqrt{2}\right) = 2(25)(2) = 100.$$

The sum of the two quarter circles in the rectangular region is

$$\begin{aligned} A &= \frac{1}{2}\pi\left(5\sqrt{2}\right)^2 \\ &= \frac{1}{2}\pi(25)(2) \\ &= 25\pi. \end{aligned}$$

The area of the shaded region is $100 - 25\pi$ square units.

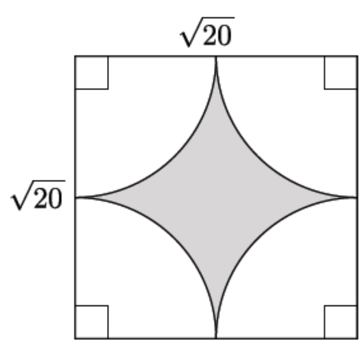
12. **Determine the exact area of the shaded region shown to the right.**

The radius of each quarter circle is $\frac{1}{2}\left(\sqrt{20}\right) = \frac{1}{2}\left(2\sqrt{5}\right) = \sqrt{5}$.

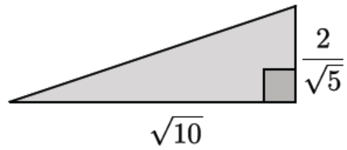
The sum of the area of the four circular regions is $A = \pi\left(\sqrt{5}\right)^2 = 5\pi$.

The area of the square is $A = \left(\sqrt{20}\right)^2 = 20$.

The area of the shaded region is $20 - 5\pi$ square units.



13. **Calculate the area of the triangle to the right.**



$$A = \frac{1}{2}(\sqrt{10})\left(\frac{2}{\sqrt{5}}\right)$$
$$= \frac{1}{2}\left(\frac{2\sqrt{10}}{\sqrt{5}}\right)$$
$$= \frac{\sqrt{10}}{\sqrt{5}}$$
$$= \sqrt{\frac{10}{5}}$$
$$= \sqrt{2}$$

The area of the triangle is $\sqrt{2}$ square units.

14. $\frac{\sqrt{2x^3}\cdot\sqrt{8x}}{\sqrt{x^3}} =$

$$\frac{\sqrt{2x^3}\cdot\sqrt{8x}}{\sqrt{x^3}} = \frac{\sqrt{16x^4}}{\sqrt{x^3}}$$
$$= \frac{4x^2}{x\sqrt{x}}$$
$$= \frac{4x}{\sqrt{x}}$$
$$= \frac{4x}{\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{x}}$$
$$= \frac{4x\sqrt{x}}{x}$$
$$= 4\sqrt{x}$$

15. **Prove rule 2 for square roots:** $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ $(a \geq 0, b > 0)$.

Let p be the nonnegative number so that $p^2 = a$, and let q be the nonnegative number so that $q^2 = b$. Then,

$$\sqrt{\frac{a}{b}} = \sqrt{\frac{p^2}{q^2}}$$ *By substitution*

$$= \sqrt{\left(\frac{p}{q}\right)^2}$$ *By the laws of exponents for integers*

$$= \frac{p}{q}$$ *By definition of square root*

$$= \frac{\sqrt{a}}{\sqrt{b}}$$ *By substitution*

Perfect Squares of Numbers 1–30

$1^2 = 1$	$16^2 = 256$
$2^2 = 4$	$17^2 = 289$
$3^2 = 9$	$18^2 = 324$
$4^2 = 16$	$19^2 = 361$
$5^2 = 25$	$20^2 = 400$
$6^2 = 36$	$21^2 = 441$
$7^2 = 49$	$22^2 = 484$
$8^2 = 64$	$23^2 = 529$
$9^2 = 81$	$24^2 = 576$
$10^2 = 100$	$25^2 = 625$
$11^2 = 121$	$26^2 = 676$
$12^2 = 144$	$27^2 = 729$
$13^2 = 169$	$28^2 = 784$
$14^2 = 196$	$29^2 = 841$
$15^2 = 225$	$30^2 = 900$