# Lesson 18: Similarity and the Angle Bisector Theorem

## Classwork

**Opening Exercise**

a.  What is an angle bisector?

b.  Describe the angle relationships formed when parallel lines are cut by a transversal.

c.  What are the properties of an isosceles triangle?

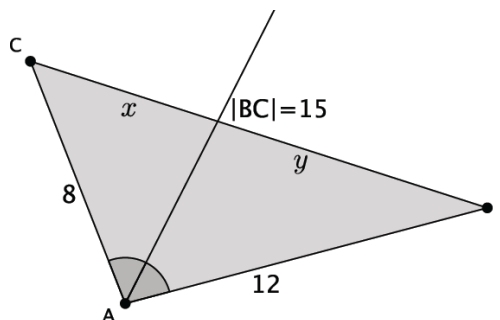## Discussion

In the diagram below, the angle bisector of $\angle A$ in $\triangle ABC$ meets side $BC$ at point $D$. Does the angle bisector create any observable relationships with respect to the side lengths of the triangle?
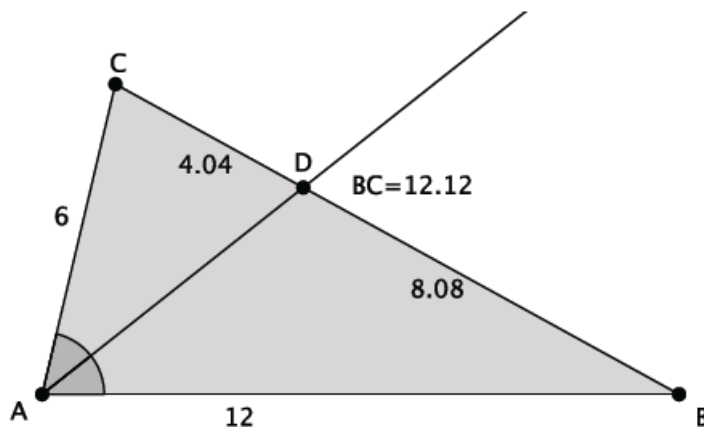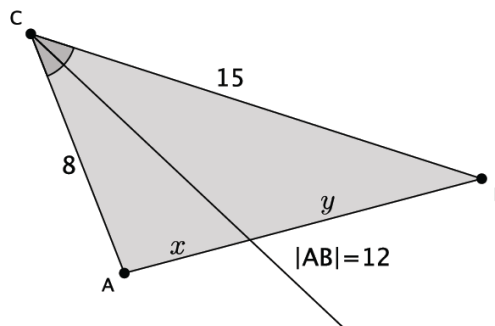
**Exercises 1–4**

1.  The sides of a triangle are 8, 12, and 15.  An angle bisector meets the side of length 15.  Find the lengths $x$ and $y$.
    Explain how you arrived at your answers.



2.  The sides of a triangle are 8, 12, and 15.  An angle bisector meets the side of length 12.  Find the lengths $x$ and $y$.

3.  The sides of a triangle are 8, 12, and 15.  An angle bisector meets the side of length 8.  Find the lengths $x$ and $y$.
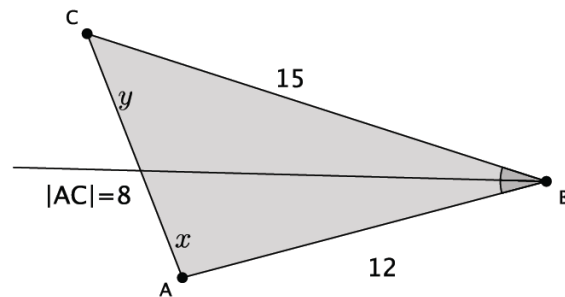


4.  The angle bisector of an angle splits the opposite side of a triangle into lengths 5 and 6.  The perimeter of the triangle is 33.  Find the lengths of the other two sides.

## Problem Set

1. The sides of a triangle have lengths of $5, 8,$ and $6\frac{1}{2}$. An angle bisector meets the side of length $6\frac{1}{2}$. Find the lengths $x$ and $y$.

2. The sides of a triangle are $10\frac{1}{2}, 16\frac{1}{2},$ and $9$. An angle bisector meets the side of length $9$. Find the lengths $x$ and $y$.

3. In the diagram of triangle $DEF$ below, $\overline{DG}$ is an angle bisector, $E = 8$, $DF = 6$, and $EF = 8\frac{1}{6}$. Find $FG$ and $EG$.
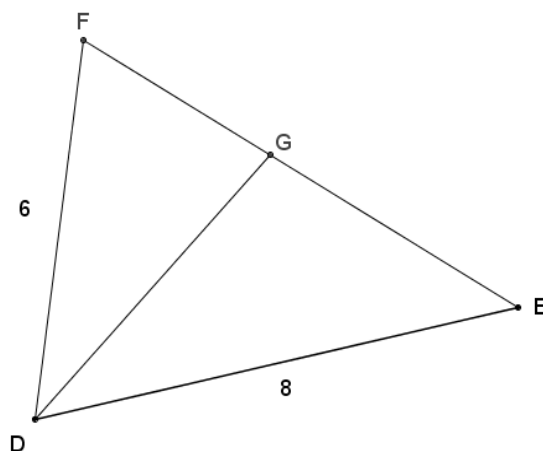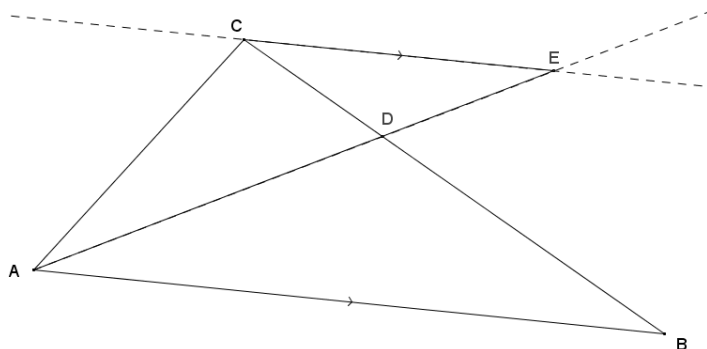


4. $\angle BAD \cong \angle DAC$, show that $BD:BA = CD:CA$.
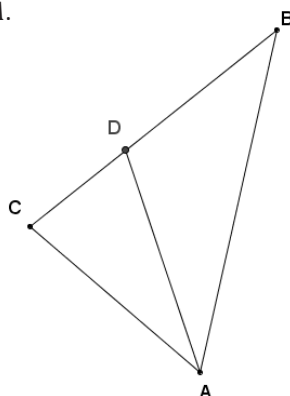


5. The perimeter of triangle $LMN$ is 32 cm. $\overline{NX}$ is the angle bisector of angle $N$, $LX = 3$ cm, and $XM = 5$ cm. Find $LN$ and $MN$.

6. Given $CD = 3$, $DB = 4$, $BF = 4$, $FE = 5$, $AB = 6$, and $\angle CAD \cong \angle DAB \cong \angle BAF \cong \angle FAE$, find the perimeter of quadrilateral $AEBC$

7. If $\overline{AE}$ meets $\overline{BC}$ at $D$ such that $CD : BD = CA : BA$, show that $\angle CAD \cong \angle BAD$. Explain how this proof relates to the angle bisector theorem.



8. In the diagram below, $\overline{ED} \cong \overline{DB}$, $\overline{BE}$ bisects $\angle ABC$, $AD = 4$, and $DC = 8$. Prove that $\triangle ADB \sim \triangle CEB$.