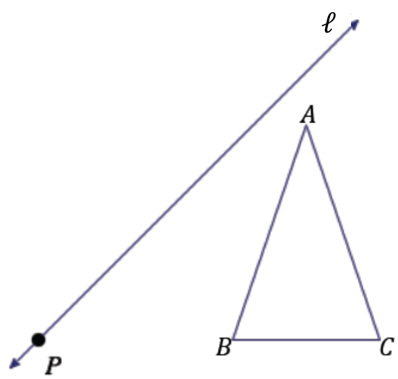
Lesson 13: Properties of Similarity Transformations

Classwork

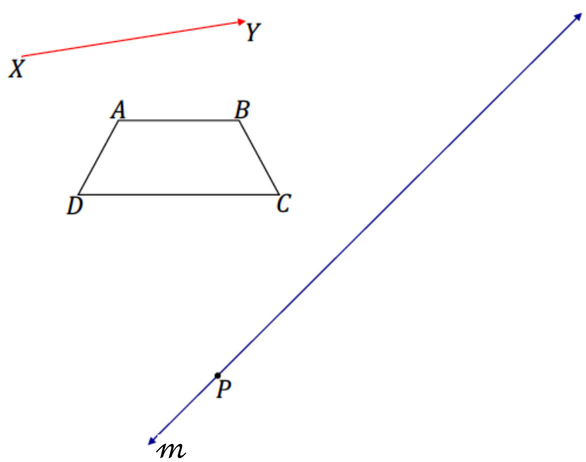
Example 1

Similarity transformation G consists of a rotation about the point P by $90°$, followed by a dilation centered at P with scale factor $r = 2$, and then a reflection across line ℓ. Find the image of the triangle.



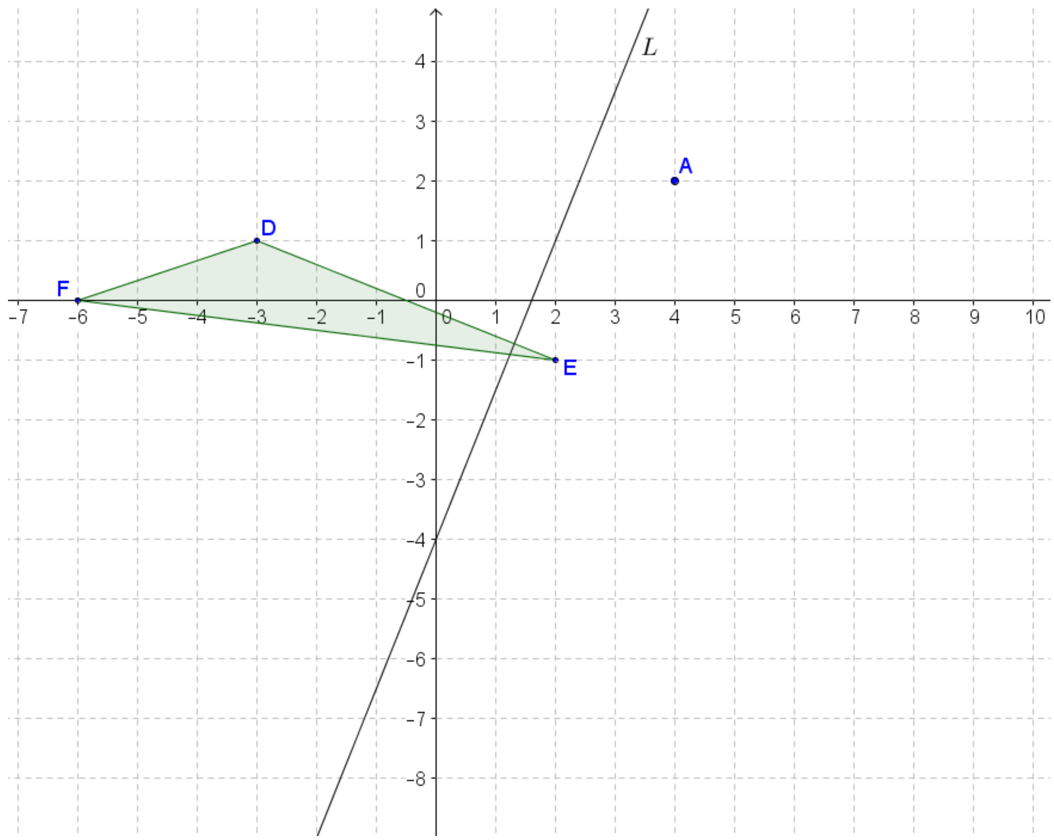
Example 2

A similarity transformation G applied to trapezoid $ABCD$ consists of a translation by vector $\overrightarrow{XY}$, followed by a reflection across line m, and then followed by a dilation centered at P with scale factor $r = 2$. Recall that we can describe the same sequence using the following notation: $D_{P,2}\left(r_m\left(T_{XY}(ABCD)\right)\right)$. Find the image of $ABCD$.



Exercise 1

A similarity transformation for triangle DEF is described by $r_n\left(D_{A,\frac{1}{2}}\left(R_{A,90^\circ}(DEF)\right)\right)$. Locate and label the image of triangle DEF under the similarity.



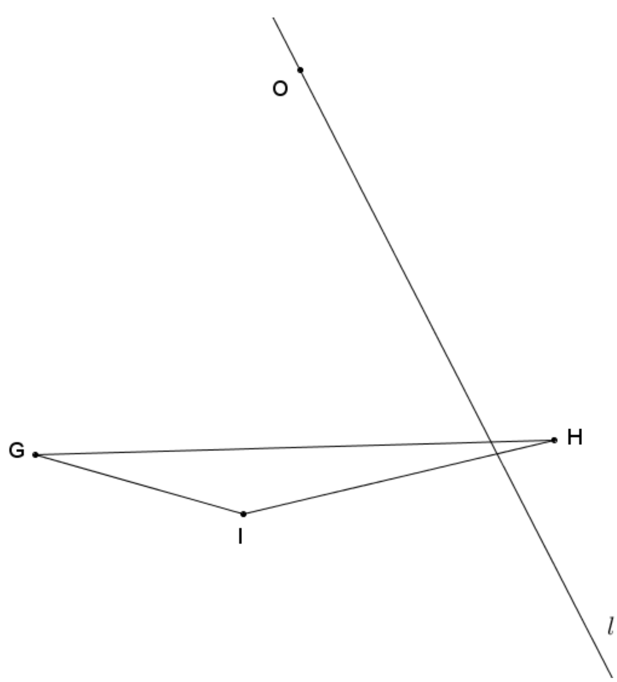
Lesson Summary

Properties of similarity transformations:

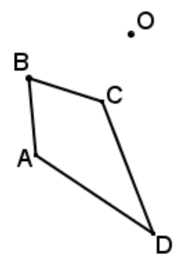
1. Distinct points are mapped to distinct points.
2. Each point P' in the plane has a pre-image.
3. There is a scale factor r for G, so that for any pair of points P and Q with images $P' = G(P)$ and $Q' = G(Q)$, then $P'Q' = rPQ$.
4. A similarity transformation sends lines to lines, rays to rays, line segments to line segments, and parallel lines to parallel lines.
5. A similarity transformation sends angles to angles of equal measure.
6. A similarity transformation maps a circle of radius R to a circle of radius rR, where r is the scaling factor of the similarity transformation.

Problem Set

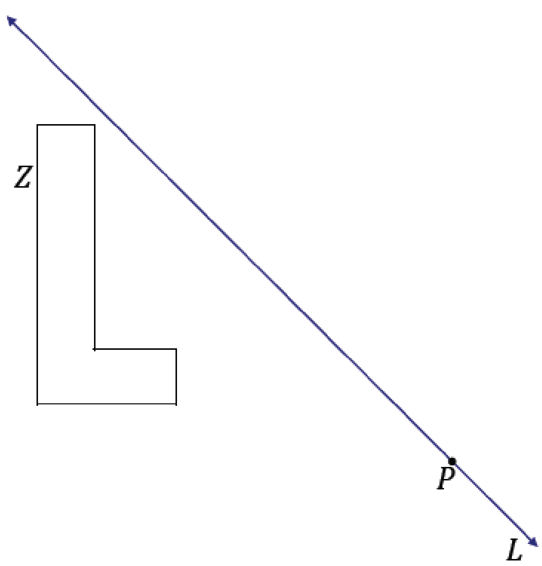
1. A similarity transformation consists of a reflection over line ℓ, followed by a dilation from O with a scale factor of $r = \frac{3}{4}$. Use construction tools to find $\triangle G''H''I''$.



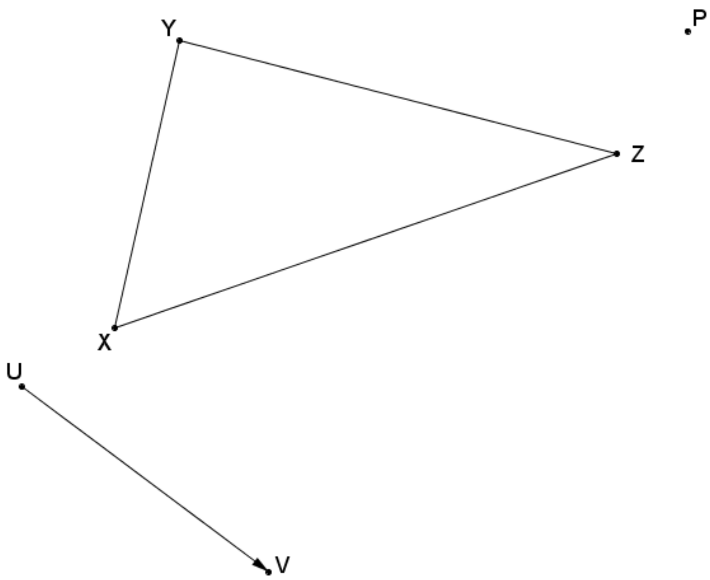
2. A similarity transformation consists of a dilation from point O with a scale factor of $r = 2\frac{1}{2}$, followed by a rotation about O of $-90°$. Use construction tools to find kite $A''B''C''D''$.



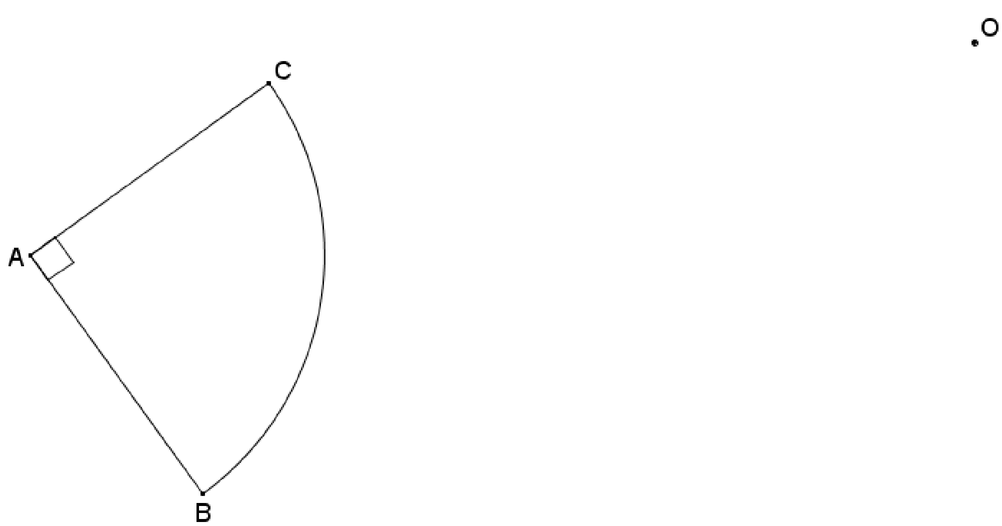
3. For the Figure Z, find the image of $r_\ell\left(R_{P,90^\circ}\left(D_{P,\frac{1}{2}}(Z)\right)\right)$.



4. A similarity transformation consists of a translation along vector $\overrightarrow{UV}$, followed by a rotation of $60°$ about P, then a dilation from P with scale factor $r = \frac{1}{3}$. Use construction tools to find $\triangle X'''Y'''Z'''$.

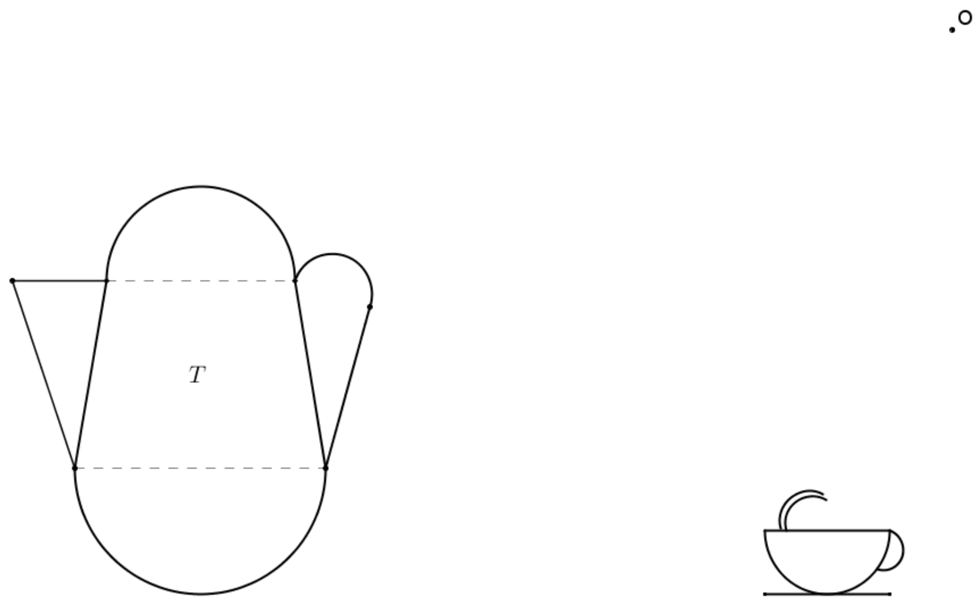


5. Given the quarter-circular figure determined by points A, B, and C, a similarity transformation consists of a $-65°$ rotation about point B, followed by a dilation from point O with a scale factor of $r = \frac{1}{2}$. Find the image of the figure determined by points A'', B'', and C''.

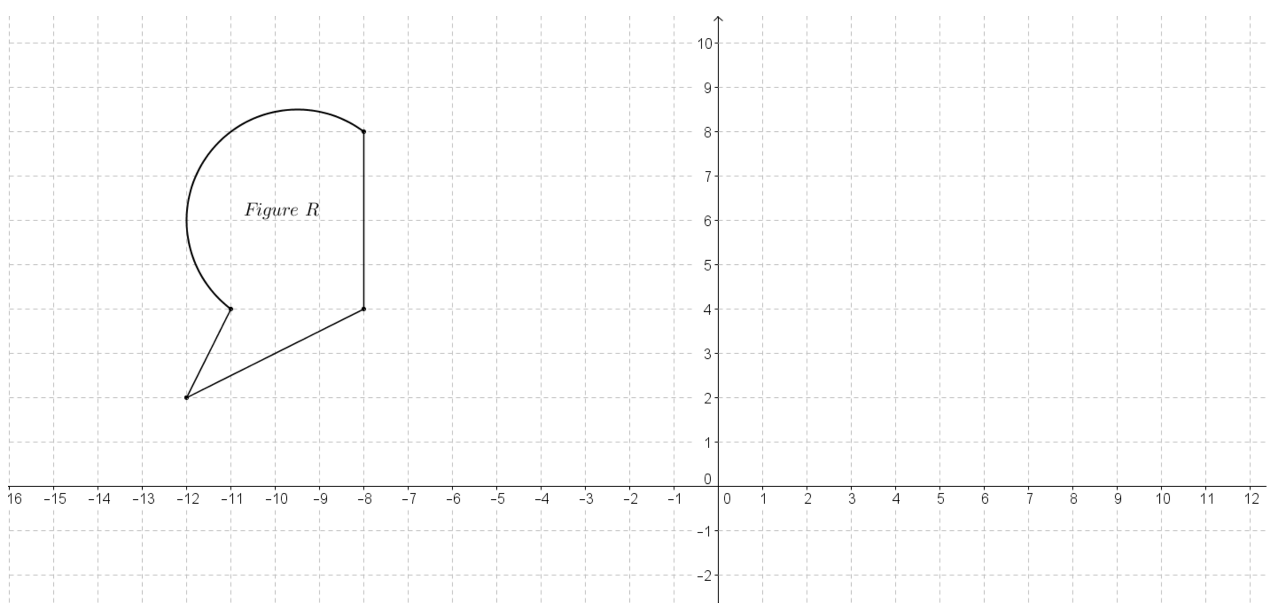


Describe a different similarity transformation that would map quarter-circle ABC to quarter-circle $A''B''C''$.

6. A similarity transformation consists of a dilation from center O with a scale factor of $\frac{1}{2}$, followed by a rotation of $60°$ about point O. Complete the similarity transformation on Figure T to complete the drawing of Figure T''.



7. Given Figure R on the coordinate plane shown below, a similarity transformation consists of a dilation from $(0,6)$ with a scale factor of $\frac{1}{4}$, followed by a reflection over line $x = -1$, then by a vertical translation of 5 units down. Find the image of Figure R.

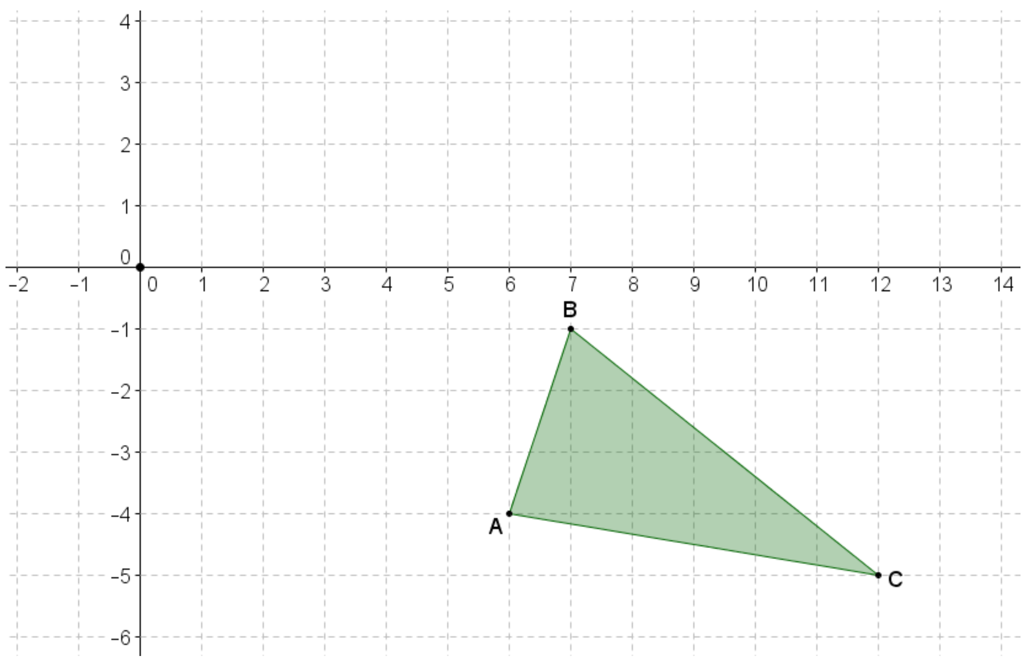


8. Given $\triangle ABC$, with vertices $A(2,-7)$, $B(-2,-1)$, and $C(3,-4)$, locate and label the image of the triangle under the similarity transformation $D_{B',\frac{1}{2}}\left(R_{A,120^\circ}\left(r_{x=2}(ABC)\right)\right)$.

9. In Problem 8, describe the relationship of A''' to $\overline{AB'}$, and explain your reasoning.

10. Given $O(-8,3)$ and quadrilateral $BCDE$, with $B(-5,1)$, $C(-6,-1)$, $D(-4,-1)$, and $E(-4,2)$, what are the coordinates of the vertices of the image of $BCDE$ under the similarity transformation $r_{x-axis}\left(D_{O,3}(BCDE)\right)$?

11. Given triangle ABC as shown on the diagram of the coordinate plane:
    a. Perform a translation so that vertex A maps to the origin.
    b. Next, dilate the image $A'B'C'$ from the origin using a scale factor of $\frac{1}{3}$.
    c. Finally, translate the image $A''B''C''$ so that the vertex A'' maps to the original point A.
    d. Using transformations, describe how the resulting image $A'''B''C''$ relates to the original figure ABC.



12.

a. In the coordinate plane, name the single transformation that is the result of the composition of the two dilations shown below:

$D_{(0,0),2}$ followed by $D_{(0,4),\frac{1}{2}}$

(Hint: Try it!)

b. In the coordinate plane, name the single transformation that is the result of the composition of the two dilations shown below:

$D_{(0,0),2}$ followed by $D_{(4,4),\frac{1}{2}}$

(Hint: Try it!)

c. Using the results from parts (a) and (b), describe what happens to the origin under both similarity transformations.