Lesson 8: How Do Dilations Map Rays, Lines, and Circles?

Student Outcomes

- Students prove that a dilation maps a ray to a ray, a line to a line, and a circle to a circle.

Lesson Notes

The objective in Lesson 7 was to prove that a dilation maps a segment to segment; in Lesson 8, students prove that a dilation maps a ray to a ray, a line to a line, and a circle to a circle. An argument similar to that in Lesson 7 can be made to prove that a ray maps to a ray; allow students the opportunity to establish this argument as independently as possible.

Classwork

Opening (2 minutes)

As in Lesson 7, remind students of their work in Grade 8, when they studied the multiplicative effect that dilation has on points in the coordinate plane when the center is at the origin. Direct students to consider what happens to the dilation of a ray that is not on the coordinate plane.

- Today we are going to show how to prove that dilations map rays to rays, lines to lines, and circles to circles.
- Just as we revisited what dilating a segment on the coordinate plane is like, so we could repeat the exercise here. How would you describe the effect of a dilation on a point (x, y) on a ray about the origin by scale factor r in the coordinate plane?
  - *A point (x, y) on the coordinate plane dilated about the origin by scale factor r would then be located at (rx, ry).*
- Of course, we must now consider what happens in the plane versus the coordinate plane.

Opening Exercise (3 minutes)

MP.3

Opening Exercise

a. **Is a dilated ray still a ray? If the ray is transformed under a dilation, explain how.**

Accept any reasonable answer. The goal of this line of questioning is for students to recognize that a segment dilates to a segment that is r times the length of the original.

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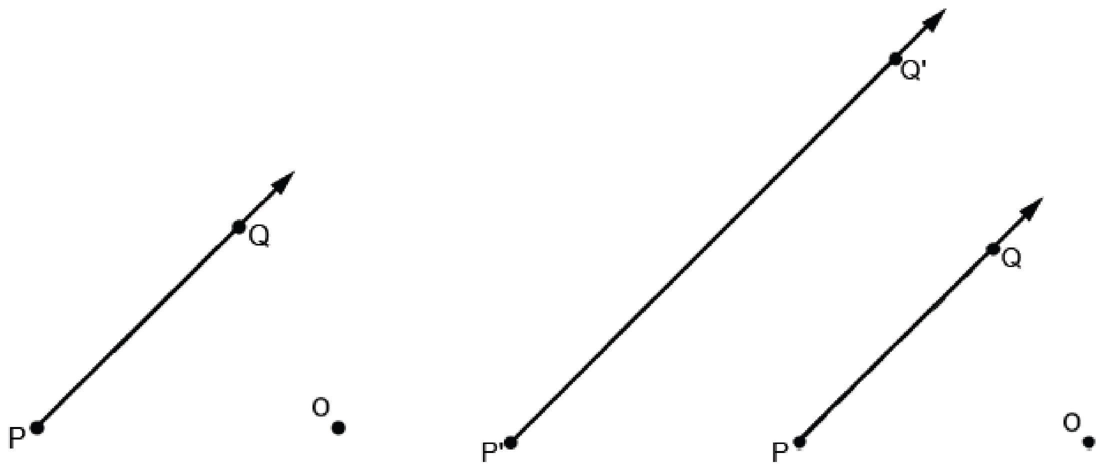
MP.3

b. Dilate the ray PQ by a scale factor of 2 from center O.



i. Is the figure $\overrightarrow{P'Q'}$ a ray?

Yes, the dilation of ray PQ produces a ray $P'Q'$.

ii. How, if at all, has the segment PQ been transformed?

The segment $P'Q'$ in ray $P'Q'$, is twice the length of the segment PQ in ray PQ. The segment has increased in length according to the scale factor of dilation.

iii. Will a ray always be mapped to a ray? Explain how you know.

Students will most likely say that a ray will always map to a ray; they may defend their answer by citing the definition of a dilation and its effect on the center and a given point.

- We are going to use our work in Lesson 7 to help guide our reasoning today.
- In proving why dilations map segments to segments, we considered three variations of the position of the center relative to the segment and value of the scale factor of dilation:
  (1) A center O and scale factor $r = 1$ and segment PQ.
  (2) A line PQ that does not contain the center O and scale factor $r \neq 1$.
  (3) A line PQ that does contain the center O and scale factor $r \neq 1$.

Point out that the condition in case (1) does not specify the location of center O relative to the line that contains segment PQ; we can tell by this description that the condition does not impact the outcome and, therefore, is not more particularly specified.

- We will use the set up of these cases to build an argument that shows dilations map rays to rays.

Examples 1–3 focus on establishing the dilation theorem of rays: A dilation maps a ray to a ray sending the endpoint to the endpoint. The arguments for Examples 1 and 2 are very similar to those of Lesson 7, Examples 1 and 3, respectively. Use discretion and student success with the Lesson 7 examples to consider allowing small-group work on the following Examples 1–2, possibly providing a handout of the solution once groups have arrived at solutions of their own. Again, since the arguments are quite similar to those of Lesson 7, Examples 1 and 3, it is important to stay within the time allotments of each example. Otherwise, Examples 1–2 should be teacher-led.

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Example 1 (2 minutes)

Encourage students to draw upon the argument from Lesson 1, Example 1. This is intended to be a quick exercise; consider giving 30-second time warnings or using a visible timer.

Example 1

Will a dilation about center O and scale factor $r = 1$ map $\overrightarrow{PQ}$ to $\overrightarrow{P'Q'}$? Explain.

A scale factor of $r = 1$ means that the ray and its image are equal. That is, the dilation does not enlarge or shrink the image of the figure but remains unchanged. Therefore, when the scale factor of dilation is $r = 1$, then the dilation maps the ray to itself.

Example 2 (6 minutes)

MP.1

Encourage students to draw upon the argument for Lesson 1, Example 3.

Example 2

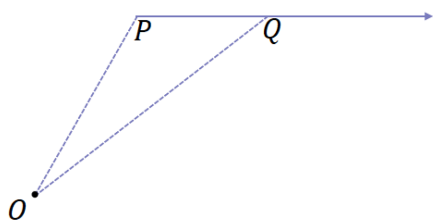
The line that contains $\overrightarrow{PQ}$ does not contain point O. Will a dilation D about center O and scale factor $r \neq 1$ map every point of $\overrightarrow{PQ}$ onto a point of $\overrightarrow{P'Q'}$?

- A restatement of this problem is: If R is a point on $\overrightarrow{PQ}$, then is $D(R)$ a point on $\overrightarrow{P'Q'}$? Also, if a point S' lies on the ray $\overrightarrow{P'Q'}$, then is there a point S on the ray $\overrightarrow{PQ}$ such that $D(S) = S'$?
- Consider the case where the center O is not in the line that contains $\overrightarrow{PQ}$, and the scale factor is $r \neq 1$. Then points O, P, and Q form $\triangle OPQ$.

Scaffolding:

- If students have difficulty following the logical progression of the proof, ask them instead to draw rays and dilate them by a series of different scale factors and then make generalization about the results.
- For students that are above grade level, ask them to attempt to prove the theorem independently.

Draw the following figure on the board.



- We examine the case with a scale factor $r > 1$; the proof for $0 < r < 1$ is similar.
- Under a dilation about center O and $r > 1$, P goes to P' and Q goes to Q'; $OP' = r \cdot OP$ and $OQ' = r \cdot OQ$. What conclusion can we draw from these lengths?
- Summarize the proof and result to a partner.

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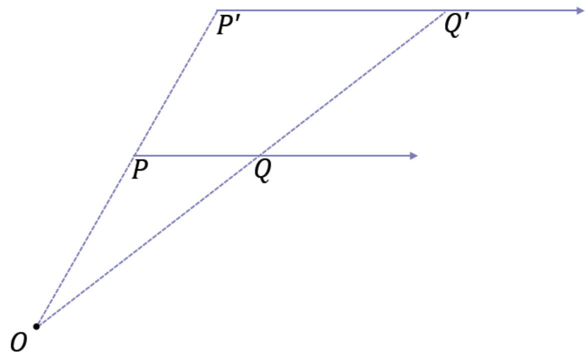
Draw the following figure on the board.

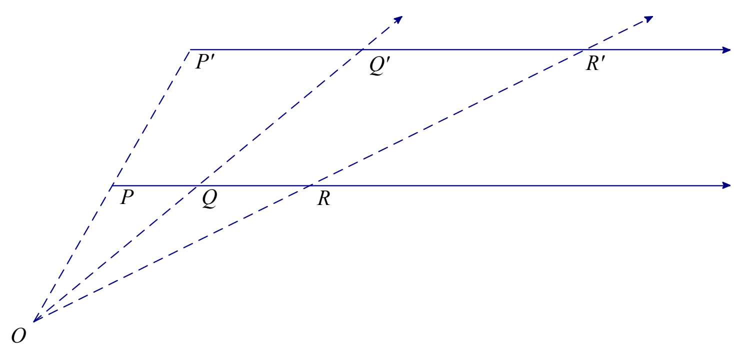


- *We can rewrite each length relationship as* $\frac{OP'}{OP} = \frac{OQ'}{OQ} = r$.

- By the triangle side splitter theorem what else can we now conclude?
  - *The segment* PQ *splits* $\triangle OP'Q'$ *proportionally.*
  - *The lines that contain* $\overrightarrow{PQ}$ *and* $\overrightarrow{P'Q'}$ *are parallel*; $\overleftrightarrow{PQ} \parallel \overleftrightarrow{P'Q'}$; therefore, $\overrightarrow{PQ} \parallel \overrightarrow{P'Q'}$.

MP.7

- By the dilation theorem for segments (Lesson 7), the dilation from O of the segment $\overline{PQ}$ is the segment $\overline{P'Q'}$, that is, $D(\overline{PQ}) = \overline{P'Q'}$ as sets of points. Therefore we need only consider an arbitrary point R on $\overrightarrow{PQ}$ that lies outside of $\overline{PQ}$. For any such point R, what point is contained in the segment $\overline{PR}$?
  - The point Q.
- Let $R' = D(R)$.



- By the dilation theorem for segments, the dilation from O of the segment $\overline{PR}$ is the segment $\overline{P'R'}$. Also by the dilation theorem for segments, the point $D(Q) = Q'$ is a point on segment $\overline{P'R'}$. Therefore the ray $\overrightarrow{P'Q'}$ and the ray $\overrightarrow{P'R'}$ must be the same ray. In particular, R' is a point on $\overrightarrow{P'Q'}$, which was what we needed to show.

To show that for every point S' on the ray $\overrightarrow{P'Q'}$ there is a point S on the ray $\overrightarrow{PQ}$ such that $D(S) = S'$, consider the dilation from center O with scale factor $\frac{1}{r}$ (the inverse of the dilation D). This dilation maps S' to a point S on the ray $\overrightarrow{PR}$ by the same reasoning as above. Then $D(S) = S'$.

- We conclude that the points of ray $\overrightarrow{PQ}$ are mapped onto to the points of ray $\overrightarrow{P'Q'}$ and, more generally, that dilations map rays to rays.

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Example 3 (12 minutes)

Encourage students to draw upon the argument for Lesson 1, Example 3.

> **Example 3**
>
> **The line that contains $\overrightarrow{PQ}$ contains point O. Will a dilation about center O and scale factor r map ray PQ to a ray $P'Q'$?**

- Consider the case where the center O belongs to the line that contains $\overrightarrow{PQ}$.

> **a. Examine the case where the endpoint P of $\overrightarrow{PQ}$ coincides with the center O of the dilation.**

- If the endpoint P of $\overrightarrow{PQ}$ coincides with the center O, what can we say about $\overrightarrow{PQ}$ and $\overrightarrow{OQ}$?

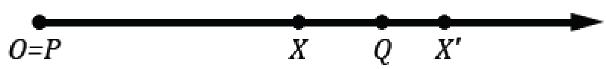
Ask students to draw what this looks like, and draw the following on the board after giving them a head start.

  - *All the points on $\overrightarrow{PQ}$ also belong to $\overrightarrow{OQ}$; $\overrightarrow{PQ} = \overrightarrow{OQ}$.*
- By definition, a dilation always sends its center to itself. What are the implications for the dilation of O and P?

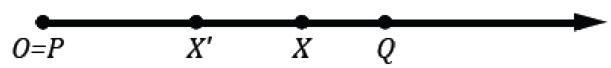
MP.1

  - *Since a dilation always sends its center to itself, then $O = P = O' = P'$.*
- Let X be a point on $\overrightarrow{OQ}$ so that $X \neq O$. What happens to X under a dilation about O with scale factor r?
  - *The dilation sends X to X' on $\overrightarrow{OX}$, and $OX' = r \cdot OX$.*

Ask students to draw what the position of X and X' might look like if $r > 1$ or $r < 1$. Points may move further away ($r > 1$) or move closer to the center ($r < 1$). We will not draw this for every case, rather, it is a reminder up front.



$r > 1$



$r < 1$

- Since O, X, and Q are collinear, then O, X', and Q will also be collinear; X' is on the ray OX, which coincides with $\overrightarrow{OQ}$ by definition of dilation.
- Therefore, a dilation of $\overrightarrow{OQ} = \overrightarrow{PQ}$ (or when the endpoint of P of $\overrightarrow{PQ}$ coincides with the center O) about center O and scale factor r maps to $\overrightarrow{O'Q'} = \overrightarrow{P'Q'}$. We have answered the bigger question that a dilation maps a ray to a ray.

> **b. Examine the case where the endpoint P of $\overrightarrow{PQ}$ is between O and Q on the line containing O, P, and Q.**

Ask students to draw what this looks like, and draw the following on the board after giving them a head start.

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- We already know from the previous case that the dilation of the ray $\overrightarrow{OQ}$ maps onto itself. All we need to show is that any point on the ray that is further away from the center than P will map to a point that is further away from the center than P'.
- Let X be a point on $\overrightarrow{PQ}$ so that $X \neq P$. What can be concluded about the relative lengths of OP and OX?

Ask students to draw what this looks like, and draw the following on the board after giving them a head start. The following is one possibility.

  - *$OP < OX$.*
- Describe how the lengths OP' and OX' compare once a dilation about center O and scale factor r sends P to P' and X to X'.
  - *$OP' = r \cdot OP$ and $OX' = r \cdot OX$.*
  - *Multiplying both sides of $OP < OX$ by $r > 0$, gives $r \cdot OP < r \cdot OX$, so $OP' < OX'$.*

MP.1

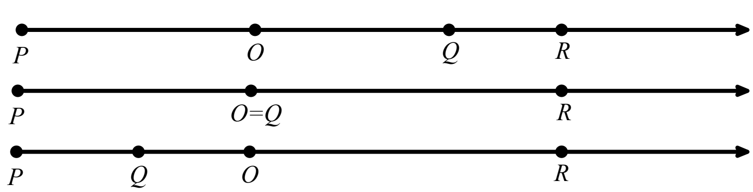
Ask students to draw what this might look like if $r > 1$ and draw the following on the board after giving them a head start.

- Therefore, we have shown that any point on the ray $\overrightarrow{PQ}$ that is further away from the center than P will map to a point that is further away from the center than P'. In this case, we still see that a dilation maps a ray to a ray.

> **c. Examine the remaining case where the center O of the dilation and point Q are on the same side of P on the line containing O, P, and Q.**

- Now consider the relative position of O and Q on $\overrightarrow{PQ}$. We will use an additional point R as a reference point so the O is between P and R.

Draw the following on the board; these are all the ways that O and Q are on the same side of P.



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MP.1

- By case (a), we know that a dilation with center O maps $\overrightarrow{OR}$ to itself.
- Also, by our work in Lesson 7 on how dilations map segments to segments, we know that $\overline{PO}$ is taken to $\overline{P'O}$, where P' lies on $\overrightarrow{OP}$.
- The union of the segment $\overline{P'O}$ and the ray $\overrightarrow{OR}$ is the ray $\overrightarrow{P'R}$. So the dialation maps the ray $\overrightarrow{PQ}$ to the ray $\overrightarrow{P'R}$.
- Since Q' is a point on the ray $\overrightarrow{P'R}$ and $Q' \neq P'$, we see $\overrightarrow{P'R} = \overrightarrow{P'Q'}$. Therefore, in this case, we still see that $\overrightarrow{PQ}$ maps to $\overrightarrow{P'Q'}$ under a dilation.

Example 4 (6 minutes)

In Example 4, students prove the dilation theorem for lines: A dilation maps a line to a line. If the center O of the dilation lies on the line or if the scale factor r of the dilation is equal to 1, then the dilation maps the line to the same line. Otherwise, the dilation maps the line to a parallel line.

The dilation theorem for lines can be proved using arguments similar to those used in Examples 1–3 for rays and to Examples 1–3 in Lesson 7 for segments. Consider asking students to prove the theorem on their own as an exercise, especially if time is an issue, and then provide the following proof.

MP.3

- We have just seen that dilations map rays to rays. How could we use this to reason that dilations map lines to lines?
  - *The line $\overleftrightarrow{PQ}$ is the union of the two rays $\overrightarrow{PQ}$ and $\overrightarrow{QP}$. Dilate rays $\overrightarrow{PQ}$ and $\overrightarrow{QP}$; the dilation yields rays $\overrightarrow{P'Q'}$ and $\overrightarrow{Q'P'}$. The line $\overleftrightarrow{P'Q'}$ is the union of the two rays $\overrightarrow{P'Q'}$ and $\overrightarrow{Q'P'}$. Since the dilation maps the rays $\overrightarrow{PQ}$ and $\overrightarrow{QP}$ to the rays $\overrightarrow{P'Q'}$ and $\overrightarrow{Q'P'}$, respectively, then the dilation maps the line $\overleftrightarrow{PQ}$ to the line $\overleftrightarrow{P'Q'}$.*

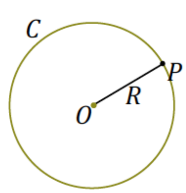
Example 5 (8 minutes)

In Example 5, students prove the dilation theorem for circles: A dilation maps a circle to a circle and maps the center to the center. Students will need the dilation theorem for circles for proving that all circles are similar in Module 5 (**G-C.A.1**).

> **Example 5**
>
> **Will a dilation about a center O and scale factor r map a circle of radius R onto another circle?**
>
> **a. Examine the case where the center of the dilation coincides with the center of the circle.**

- We first do the case where the center of the dilation is also the center of the circle. Let C be a circle with center O and radius R.

Draw the following figure on the board.



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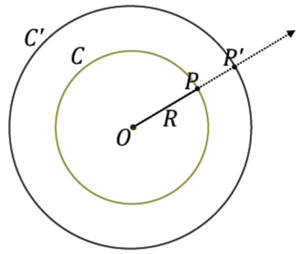
- If the center of the dilation is also O, then every point P on the circle is sent to a point P' on $\overrightarrow{OP}$ so that $OP' = r \cdot OP = rR$; i.e., the point goes to the point P' on the circle C' with center O and radius rR.
- We also need to show that every point on C' is the image of a point from C: For every point P' on circle C', put a coordinate system on the line $\overleftrightarrow{OP'}$ such that the ray $\overrightarrow{OP'}$ corresponds to the nonnegative real numbers with zero corresponding to point O (by the ruler axiom). Then there exists a point P on $\overrightarrow{OP'}$ such that $OP = R$, that is, P is a point on the circle C that is mapped to P' by the dilation.

Draw the following figure on the board.

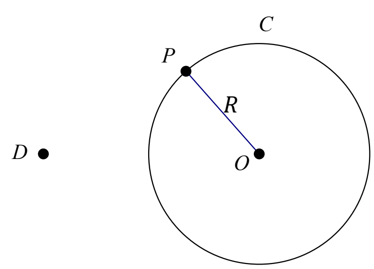


- Effectively, a dilation moves every point on a circle toward or away from the center the same amount, so the dilated image is still a circle. Thus, the dilation maps the circle C to the circle C'.
- Circles that share the same center are called *concentric circles.*

b. Examine the case where the center of the dilation is not the center of the circle; we call this the general case.

- The proof of the general case works no matter where the center of dilation is. We can actually use this proof for case (a), when the center of the circle coincides with the center of dilation.
- Let C be a circle with center O and radius R. Consider a dilation with center D and scale factor r that maps O to O'. We will show that the dilation maps the circle C to the circle C' with center O' and radius rR.

Draw the following figure on the board.



- If P is a point on circle C and the dilation maps P to P', the dilation theorem implies that $O'P' = rOP = rR$. So P' is on circle C'.
- We also need to show that every point of C' is the image of a point from C. There are a number of ways to prove this, but we will follow the same idea that we used in part (a). For a point P' on circle C' that is not on line $\overleftrightarrow{DO'}$, consider the ray $\overrightarrow{O'P'}$ (the case when P' is on line $\overleftrightarrow{DO'}$ is straightforward). Construct line ℓ through O such that $\ell \parallel \overrightarrow{O'P'}$, and let A be a point on ℓ that is in the same half-plane of $\overleftrightarrow{DO'}$ as P'. Put a coordinate system on ℓ such that the ray $\overrightarrow{OA}$ corresponds to the nonnegative numbers with zero corresponding to point O

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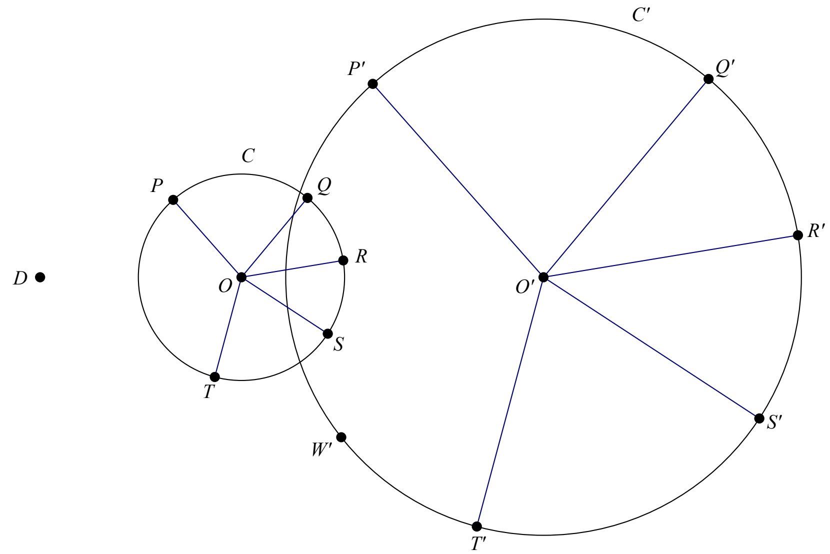
(by the ruler axiom). Then there exists a point P on $\overrightarrow{OA}$ such that $OP = R$, which implies that P is on the circle C. By the dilation theorem, P is mapped to the point P' on the circle C'.

- The diagram below shows how the dilation maps points P, Q, R, S, and T of circle C. Ask students to find point W on circle C that is mapped to point W' on circle C'.



Closing (1 minute)

Ask students to respond to the following questions and summarize the key points of the lesson.

- How are the proofs for the dilation theorems on segments, rays, and lines similar to each other?

Theorems addressed in this lesson:

- **DILATION THEOREM FOR RAYS**: A dilation maps a ray to a ray sending the endpoint to the endpoint.
- **DILATION THEOREM FOR LINES**: A dilation maps a line to a line. If the center O of the dilation lies on the line or if the scale factor r of the dilation is equal to 1, then the dilation maps the line to the same line. Otherwise, the dilation maps the line to a parallel line.
- **DILATION THEOREM FOR CIRCLES**: A dilation maps a circle to a circle and maps the center to the center.

Lesson Summary

- **DILATION THEOREM FOR RAYS: A dilation maps a ray to a ray sending the endpoint to the endpoint.**
- **DILATION THEOREM FOR LINES: A dilation maps a line to a line. If the center O of the dilation lies on the line or if the scale factor r of the dilation is equal to 1, then the dilation maps the line to the same line. Otherwise, the dilation maps the line to a parallel line.**
- **DILATION THEOREM FOR CIRCLES: A dilation maps a circle to a circle, and maps the center to the center.**

Exit Ticket (5 minutes)

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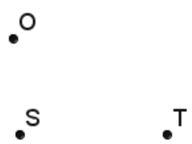
Name ____ Date ____________________

Lesson 8: How Do Dilations Map Rays, Lines, and Circles?

Exit Ticket

Given points O, S, and T below, complete parts (a)–(e):



a. Draw rays $\overrightarrow{ST}$ and $\overrightarrow{TS}$. What is the union of these rays?

b. Dilate $\overrightarrow{ST}$ from O using scale factor $r = 2$. Describe the image of $\overrightarrow{ST}$.

c. Dilate $\overrightarrow{TS}$ from O using scale factor $r = 2$. Describe the image of $\overrightarrow{TS}$.

d. What does the dilation of the rays in parts (b) and (c) yield?

e. Dilate circle C with radius TS from O using scale factor $r = 2$.

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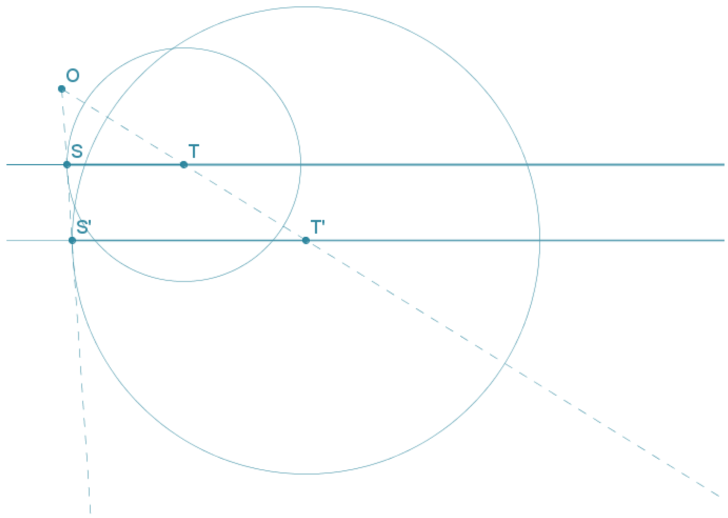
Exit Ticket Sample Solutions

Given points O, S, and T below, complete parts (a)–(e):



a. Draw rays $\overrightarrow{ST}$ and $\overrightarrow{TS}$. What is the union of these rays?

The union of $\overrightarrow{ST}$ and $\overrightarrow{TS}$ is line ST.

b. Dilate $\overrightarrow{ST}$ from O using scale factor $r = 2$. Describe the image of $\overrightarrow{ST}$.

The image of $\overrightarrow{ST}$ is $\overrightarrow{S'T'}$.

c. Dilate $\overrightarrow{TS}$ from O using scale factor $r = 2$. Describe the image of $\overrightarrow{TS}$.

The image of $\overrightarrow{TS}$ is $\overrightarrow{T'S'}$.

d. What does the dilation of the rays in parts (b) and (c) yield?

The dilation of rays $\overrightarrow{ST}$ and $\overrightarrow{TS}$ yields $\overleftrightarrow{S'T'}$.

e. Dilate circle T with radius TS from O using scale factor $r = 2$.

See diagram above.

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Problem Set Sample Solutions

1. In Lesson 8, Example 2, you proved that a dilation with a scale factor $r > 1$ maps a ray PQ to a ray $P'Q'$. Prove the remaining case that a dilation with scale factor $0 < r < 1$ maps a ray PQ to a ray $P'Q'$.

   Given the dilation $D_{O,r}$, with $0 < r < 1$ maps P to P' and Q to Q', prove that $D_{O,r}$ maps $\overrightarrow{PQ}$ to $\overrightarrow{P'Q'}$.

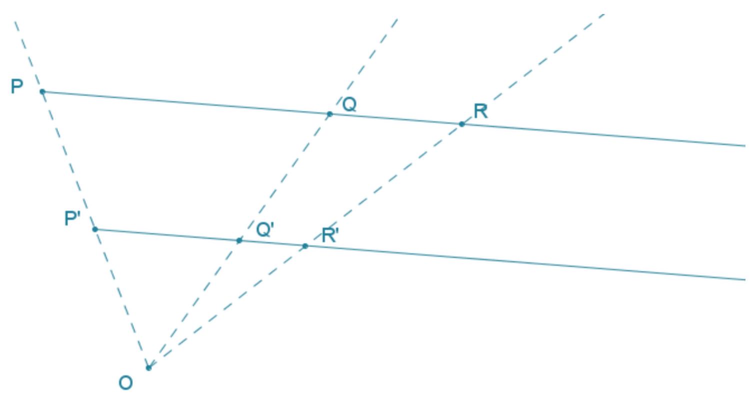
   By the definition of dilation, $OP' = r(OP)$, and likewise, $\frac{OP'}{OP} = r$.

   By the dilation theorem, $\overline{P'Q'} \parallel \overline{PQ}$.

   Through two different points lies only one line, so $\overleftrightarrow{PQ} \parallel \overleftrightarrow{P'Q'}$.

   Draw point R on $\overrightarrow{PQ}$ and then draw $\overrightarrow{OR}$. Mark point R' at intersection of $\overrightarrow{OR}$ and $\overrightarrow{P'Q'}$.

   By the triangle side splitter theorem, $\overline{P'R'}$ splits $\overline{OP}$ and $\overline{OR}$ proportionally, so $\frac{OR'}{OR} = \frac{OP'}{OP} = r$. Therefore, because R was chosen as an arbitrary point, $OR' = r(OR)$ for any point R on $\overrightarrow{PQ}$.

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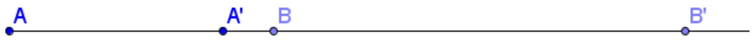
2. In the diagram below, $\overrightarrow{A'B'}$ is the image of $\overrightarrow{AB}$ under a dilation from point O with an unknown scale factor, A maps to A' and B maps to B'. Use direct measurement to determine the scale factor r, and then find the center of dilation O.

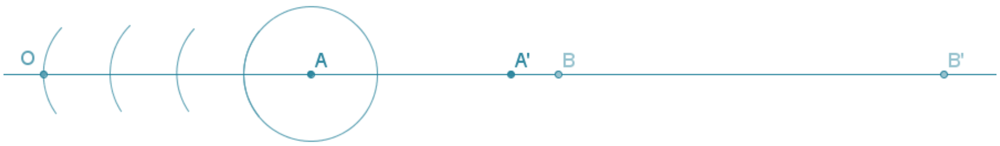


By the definition of dilation, $A'B' = r\,(AB)$, $OA' = r(OA)$, and $OB' = r(OB)$. By direct measurement,
$\frac{A'B'}{AB} = \frac{7}{4} = r$.

The images of A and B are pushed to the right on $\overleftrightarrow{AB}$ under the dilation and $A'B' > AB$, so the center of dilation must lie on $\overleftrightarrow{AB}$ to the left of points A and B.

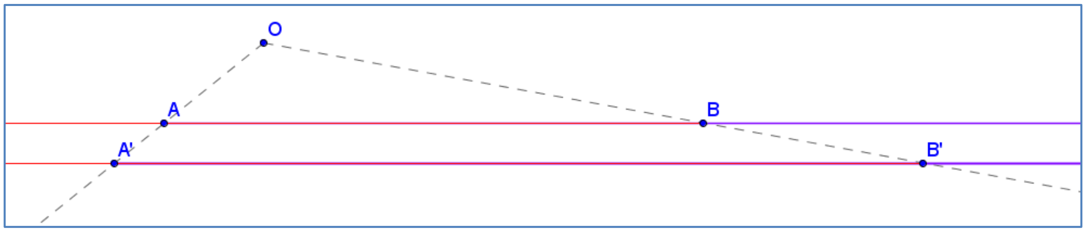
By the definition of dilation:

$$\begin{aligned} OA' &= \frac{7}{4}(OA) \\ (OA + AA') &= \frac{7}{4}(OA) \\ \frac{OA + AA'}{OA} &= \frac{7}{4} \\ \frac{OA}{OA} + \frac{AA'}{OA} &= \frac{7}{4} \\ 1 + \frac{AA'}{OA} &= \frac{7}{4} \\ \frac{AA'}{OA} &= \frac{3}{4} \\ AA' &= \frac{3}{4}(OA) \\ \frac{4}{3}(AA') &= OA \end{aligned}$$



3. Draw a line $\overleftrightarrow{AB}$ and dilate points A and B from center O where O is not on $\overleftrightarrow{AB}$. Use your diagram to explain why a line maps to a line under a dilation with scale factor r.

Two rays $\overrightarrow{AB}$ and $\overrightarrow{BA}$ on the points A and B point in opposite directions as shown in the diagram below. The union of the two rays is $\overleftrightarrow{AB}$. We showed that a ray maps to a ray under a dilation, so $\overrightarrow{AB}$ maps to $\overrightarrow{A'B'}$; likewise, $\overrightarrow{BA}$ maps to $\overrightarrow{B'A'}$. The dilation yields two rays $\overrightarrow{A'B'}$ and $\overrightarrow{B'A'}$ on the points A' and B' pointing in opposite directions. The union of the two rays is $\overleftrightarrow{A'B'}$; therefore, it is true that a dilation maps a line to a line.



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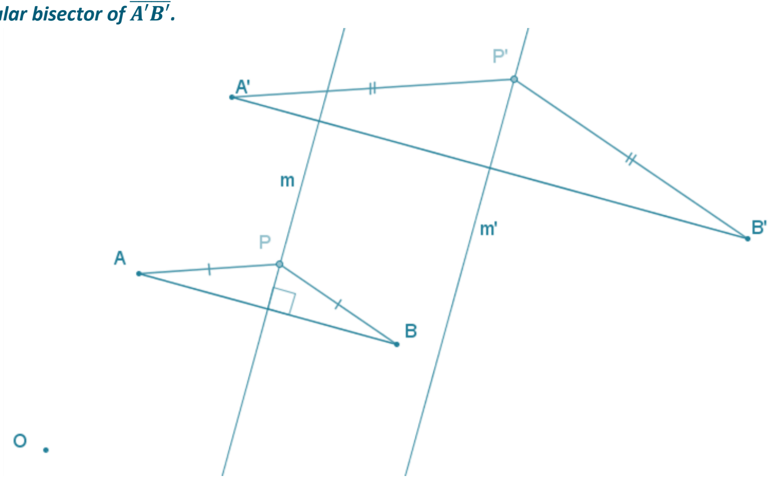
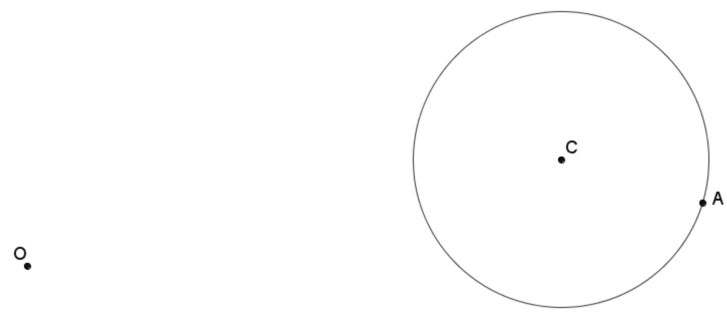
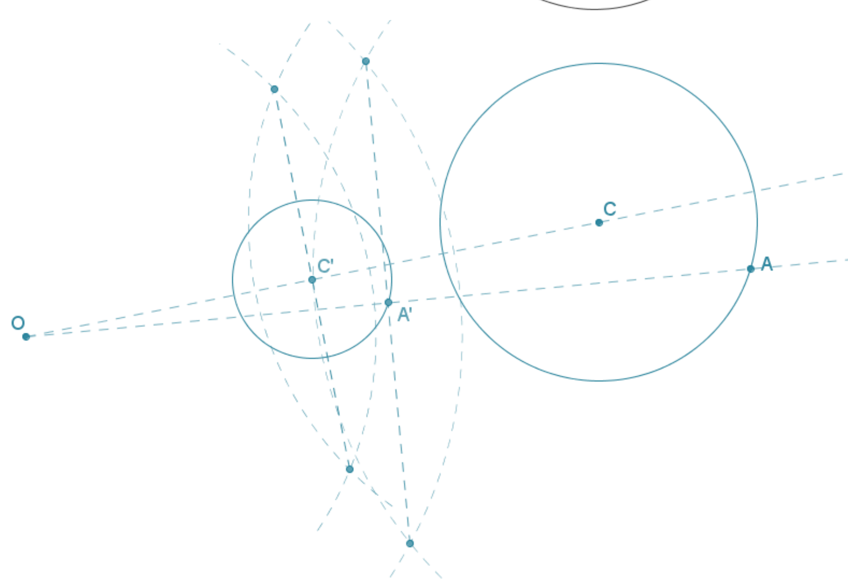
4. Let $\overline{AB}$ be a line segment, and let m be a line that is the perpendicular bisector of $\overline{AB}$. If a dilation with scale factor r maps $\overline{AB}$ to $\overline{A'B'}$ (sending A to A' and B to B') and also maps line m to line m', show that m' is the perpendicular bisector of $\overline{A'B'}$.

Let P be a point on line m and let the dilation send P to the point P' on line m'. Since P is on the perpendicular bisector of $\overline{AB}$, $PA = PB$. By the dilation theorem, $P'A' = rPA$ and $P'B' = rPB$. So $P'A' = P'B'$ and P' is on the perpendicular bisector of $\overline{A'B'}$.



5. Dilate circle C with radius CA from center O with a scale factor $r = \frac{1}{2}$.





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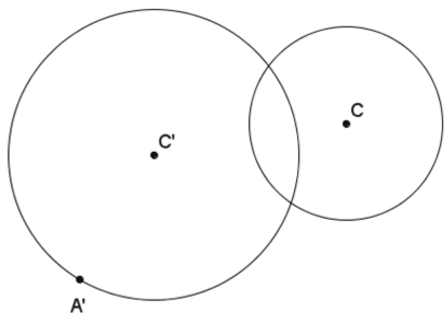
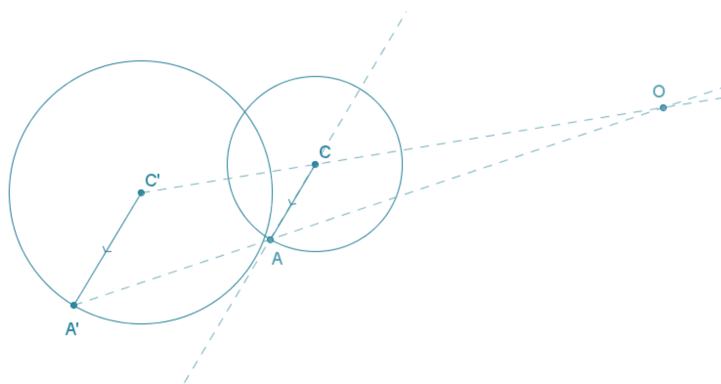
6. **In the picture below, the larger circle is a dilation of the smaller circle. Find the center of dilation O.**



Draw $\overrightarrow{C'C}$. Center O lies on $\overrightarrow{C'C}$. Since A' is not on a line with both C and C', I can use the parallel method to find point A on circle C such that $\overleftrightarrow{CA} \parallel \overleftrightarrow{C'A'}$.

Under a dilation, a point and its image(s) lie on a ray with endpoint O, the center of dilation. Draw $\overrightarrow{A'A}$ and label center of dilation $O = \overrightarrow{C'C} \cap \overrightarrow{A'A}$.



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