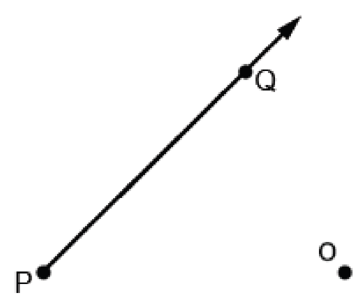
Lesson 8: How Do Dilations Map Rays, Lines, and Circles?

Classwork

Opening Exercise

a. Is a dilated ray still a ray? If the ray is transformed under a dilation, explain how.

b. Dilate the ray $\overrightarrow{PQ}$ by a scale factor of 2 from center O.



i. Is the figure $\overrightarrow{PQ}$ a ray?

ii. How, if at all, has the segment $\overrightarrow{PQ}$ been transformed?

iii. Will a ray always be mapped to a ray? Explain how you know.

Example 1

Will a dilation about center O and scale factor $r = 1$ map $\overrightarrow{PQ}$ to $\overrightarrow{P'Q'}$? Explain.

Example 2

The line that contains $\overrightarrow{PQ}$ does not contain point O. Will a dilation about center O and scale factor $r \neq 1$ map $\overrightarrow{PQ}$ to $\overrightarrow{P'Q'}$?

Example 3

The line that contains $\overrightarrow{PQ}$ contains point O. Will a dilation about center O and scale factor r map ray PQ to a ray $P'Q'$?

a. Examine the case where the endpoint P of $\overrightarrow{PQ}$ coincides with the center O of the dilation.

b. Examine the case where the endpoint P of $\overrightarrow{PQ}$ is between O and Q on the line containing O, P, and Q.

c. Examine the remaining case where the center O of the dilation and point Q are on the same side of P on the line containing O, P, and Q.

Example 5

Will a dilation about a center O and scale factor r map a circle of radius R onto another circle?

a. Examine the case where the center of the dilation coincides with the center of the circle.

b. Examine the case where the center of the dilation is not the center of the circle; we call this the general case.

Lesson Summary

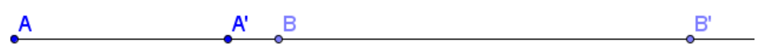
- **DILATION THEOREM FOR RAYS**: A dilation maps a ray to a ray sending the endpoint to the endpoint.
- **DILATION THEOREM FOR LINES**: A dilation maps a line to a line. If the center O of the dilation lies on the line or if the scale factor r of the dilation is equal to 1, then the dilation maps the line to the same line. Otherwise, the dilation maps the line to a parallel line.
- **DILATION THEOREM FOR CIRCLES**: A dilation maps a circle to a circle, and maps the center to the center.

Problem Set

1. In Lesson 8, Example 2, you proved that a dilation with a scale factor $r > 1$ maps a ray PQ to a ray $P'Q'$. Prove the remaining case that a dilation with scale factor $0 < r < 1$ maps a ray PQ to a ray $P'Q'$.

   Given the dilation $D_{O,r}$, with $0 < r < 1$ maps P to P' and Q to Q', prove that $D_{O,r}$ maps $\overrightarrow{PQ}$ to $\overrightarrow{P'Q'}$.

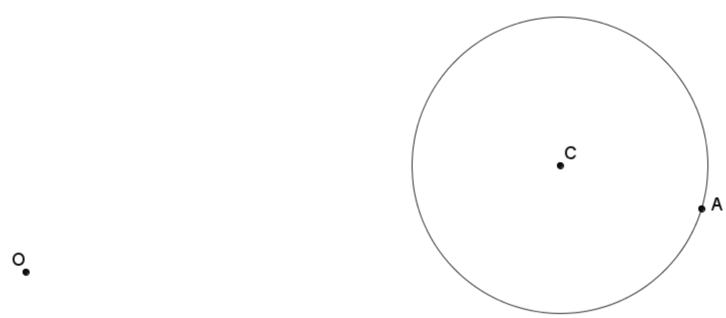
2. In the diagram below, $\overrightarrow{A'B'}$ is the image of $\overrightarrow{AB}$ under a dilation from point O with an unknown scale factor, A maps to A' and B maps to B'. Use direct measurement to determine the scale factor r, and then find the center of dilation O.



3. Draw a line AB and dilate points A and B from center O where O is not on $\overleftrightarrow{AB}$. Use your diagram to explain why a line maps to a line under a dilation with scale factor r.

4. Let $\overline{AB}$ be a line segment, and let m be a line that is the perpendicular bisector of $\overline{AB}$. If a dilation with scale factor r maps $\overline{AB}$ to $\overline{A'B'}$ (sending A to A' and B to B') and also maps line m to line m', show that m' is the perpendicular bisector of $\overline{A'B'}$.

5. Dilate circle C with radius CA from center O with a scale factor $r = \frac{1}{2}$.



6. In the picture below, the larger circle is a dilation of the smaller circle. Find the center of dilation O.