Lesson 24: Differences Due to Random Assignment Alone

Student Outcomes

- Students understand that when one group is randomly divided into two groups, the two groups' means will differ just by chance (a consequence of the random division).
- Students understand that when one group is randomly divided into two groups, the distribution of the difference in the two groups' means can be described in terms of shape, center, and spread.

Lesson Notes

This lesson investigates differences in group means when a single group is randomly divided into two groups. The goal of this lesson is for students to understand that when a single group is randomly divided into two groups, the two group means will tend to differ just by chance. Students are given 20 values which they randomly divide into two groups. The mean is then calculated for each group. The process is repeated two more times, and all group means are used to create a class dot plot, which confirms that the distribution of the random groups' means will be centered at the single set's mean. This idea is fundamental to the lessons that follow, which involve distinguishing meaningful differences in means from differences that might be due only to chance.

Classwork

This lesson is designed for students to work individually. However, students can discuss some answers with their neighbor before a class discussion of the answers. Prior to class, make a copy of Appendix A for each student. Scissors may also be needed to cut the table into pieces.

Exercises 1-17 (40–45 minutes)

Read the scenario regarding the fastest speeds driven by twenty adult drivers. You may wish (but it is not necessary) to talk about the data before beginning the lesson.

Optional questions:

- One driver answered that the fastest speed he/she had driven was 40 mph. How is this possible?
  - *This person may be learning how to drive and is still unsure of his/her driving abilities.*
- Another driver answered that the fastest speed he/she had driven was 110 mph. How is this possible?
  - *This person may have driven on a racetrack.*
- What would it mean if a driver's response was 0?
  - *This person may have never driven a car.*

Allow about 5 minutes for students to answer Exercises 1 and 2.

Exercises 1–17

Twenty adult drivers were asked the following question:

"What speed is the fastest that you have driven?"

The table below summarizes the fastest speeds driven in miles per hour (mph).

70	60	70	95	50	60	80	75	55	90
110	65	65	65	55	70	75	70	65	40

1. **What is the mean fastest speed driven?**

   69.25 mph

2. **What is the range of fastest speed driven?**

   70 mph

Scaffolding:

- Include a visual of a car and discuss the meaning of the question.
- Review the concepts of mean and range, model the process for determining these in the example, and discuss their meaning in context.

Next, students will investigate what happens to the mean fastest speed driven when the original 20 values are randomly divided into two groups. Give each student a copy of Appendix A and let them independently answer Exercise 3.

MP.3

3. **Imagine that the fastest speeds were randomly divided into two groups. How would the means and ranges compare to one another? To the means and ranges of the whole group? Explain your thinking.**

   Answers will vary. Sample response – I think the mean of each group will be fairly close because most of the values appear to be between 60 and 80. I don't think the ranges of each group will be the same. Depending on the values in the group, one possible range could be as large as 70 mph (from 40 to 110) or as small as 10 mph (from 60 to 70). I think the means of the two groups should be close to the mean of the whole group, again, because most values appear to be between 60 and 80. The range of the whole group is 70 mph, so the range of the two groups may be smaller than that.

Scaffolding:

Advanced students may be encouraged to develop their own plan for investigating the answer to the question of how the means of randomly selected groups are related to one another. Consider allowing them time to write, carry out, and evaluate plans for exploring this question.

Allow about 5 minutes for students to answer Exercises 4 and 5. Discuss the students' answers to Exercise 5.

Instructions for randomly dividing the twenty fastest speeds driven into two groups:

1. Cut (or tear) along the lines in the table so that 20 equal slips of paper are obtained.
2. Turn the squares upside down. Mix well.
3. Separate the squares into 2 piles of 10, identifying one pile as Group 1 and the other pile as Group 2.
4. Turn the slips of paper over.
5. Record the numbers for Group 1 and Group 2 in the table (Exercise 4).

Let's investigate what happens when the fastest speeds driven are randomly divided into two equal-size groups.

4. Following the instructions from your teacher, randomly divide the 20 values in the table above into two groups of 10 values each.

Sample student responses – answers will vary. One example follows.

											Mean
Group 1	70	40	65	75	65	70	65	110	60	55	
Group 2	75	70	65	60	70	95	50	55	90	80	

5. Do you expect the means of these two groups to be equal? Why or why not?

The values of the means of the two groups will probably not be exactly equal, but I don't expect them to be very different.

Allow students about 5 minutes to answer Exercises 6–8. Then, discuss answers as a class.

Sample student responses – answers will vary. Sample responses are based on the sample answer provided for Exercise 4.

6. Compute the means of these two groups. Write the means in the chart above.

Group 1 mean = 67.5 mph, Group 2 mean = 71 mph

7. How do these two means compare to each other?

The values of these two means are not very different from each other.

8. How do these two means compare to the mean fastest speed driven for the entire group (Exercise 1)?

The values of these two means are close to the original mean. The value of one mean is larger than the original mean, and the value of the other mean is smaller than the original mean. (Note: The two group mean values are equidistant from the original mean value.)

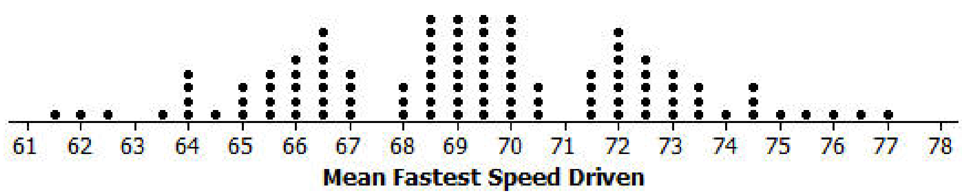
Allow about 15 minutes for students to answer Exercises 9 and 10. For the class dot plot in Exercise 10, create a number line on a whiteboard or paper. The horizontal scale should range from 61 mph to 78 mph, using tick marks of 0.5. The horizontal label should be "Mean Fastest Speed Driven." Provide 2 or 3 markers so that multiple students can place their means on the graph at the same time.

Sample student responses – answers will vary. Sample answers are shown here.

9. **Use the instructions provided for Exercise 4 to repeat the random division process two more times. Compute the mean of each group for each of the random divisions into two groups. Record your results in the tables below.**

											Mean
Group 3	55	60	95	70	75	50	65	55	70	75	67
Group 4	65	110	70	65	70	65	40	90	80	60	71.5
Group 5	65	70	55	65	75	70	75	70	80	70	69.5
Group 6	60	65	50	65	95	60	110	40	90	55	69

10. **Plot the means of all six groups on a class dot plot.**



When the class dot plot is finished, allow students about 3 minutes to answer Exercise 11. Discuss the answer as a class.

11. **Based on the class dot plot, what can you say about the possible values of the group means?**

Answers will vary.

The group sample means are centered at the original mean of 69.25 *mph.*

There is variability in the group means. Some groups' means vary more from the original mean of 69.25 *than others.*

The group means that are closer to the original mean of 69.25 *mph occur more often than the group means that are further away from* 69.25 *mph.*

Allow about 5 minutes for students to answer Exercises 12–14. Then, discuss answers as a class.

12. **What is the smallest possible value for a group mean? Largest possible value?**

Smallest possible mean (the 10 *smallest values) =* $\dfrac{40+50+55+55+60+60+65+65+65+65}{10} = 58$ *mph.*

Largest possible mean (the 10 *largest values) =* $\dfrac{70+70+70+70+75+75+80+90+95+110}{10} = 80.5$ *mph.*

13. **What is the largest possible range for the distribution of group means?**

From Exercise 12, the largest possible range is $80.5 - 58 = 22.5$ *mph.*

14. **How does the largest possible range in the group means compare to the range of the original data set (Exercise 2)? Why is this so?**

    The range of the original data is 70 mph. The largest possible range for the distribution of group means is 22.5 mph, which is much smaller. This difference is due to the use of means. The means of the two groups of 10 don't vary as much as the individual observations in the data set.

Allow students about 3 minutes to answer Exercises 15–16. Discuss these answers as a class. Anticipate that students might need help with Exercise 16.

15. **What is the shape of the distribution of group means?**

    It is symmetrical.

16. **Will your answer to the above question always be true? Explain.**

    Yes. When a single set of values is divided into two equal groups, the two group means will be equidistant from the single set's mean. Thus, it will always produce a symmetrical distribution.

Allow students about 3 minutes to answer Exercise 17 independently or with a partner. Discuss this answer.

MP.2

17. **When a single set of values is randomly divided into two equal groups, explain how the means of these two groups may be very different from each other and may be very different from the mean of the single set of values.**

    It is possible that the random division could result in most of the smaller values being in one group and most of the larger values in the other group. This would produce group means that were very different from each other and from the single set's mean. But with a random division, this is not very likely to occur.

Closing (2 minutes)

MP.3

- Compare your conjecture about the randomized groups (Exercise 3) with what you have learned.

Ask students to summarize the main ideas of the lesson in writing or with a neighbor. Use this as an opportunity to informally assess comprehension of the lesson. The Lesson Summary below offers some important ideas that should be included.

Lesson Summary

When a single set of values is randomly divided into two groups,

- **The two group means will tend to differ just by chance.**
- **The distribution of random groups' means will be centered at the single set's mean.**
- **The range of the distribution of the random groups' means will be smaller than the range of the data set.**
- **The shape of the distribution of the random groups' means will be symmetrical.**

Exit Ticket (3 minutes)

Name ____ Date ____________________

Lesson 24: Differences Due to Random Assignment Alone

Exit Ticket

When a single group is randomly divided into two groups, why do the two group means tend to be different?

Exit Ticket Sample Solutions

When a single group is randomly divided into two groups, why do the two group means tend to be different?

The two group means tend to be different because of random chance involved in the process of dividing the original group into two subgroups.

Problem Set Sample Solutions

In one high school, there are eight math classes during 2nd period. The number of students in each 2nd period math class is recorded below.

$$32 \quad 27 \quad 26 \quad 23 \quad 25 \quad 22 \quad 30 \quad 19$$

This data set is randomly divided into two equal size groups, and the group means are computed.

1. **Will the two groups means be the same? Why or why not?**

   No, the two group means tend to not be the same just due to chance.

The random division into two groups process is repeated many times to create a distribution of group mean class size.

2. **What is the center of the distribution of group mean class size?**

   The distribution of the group mean class size is centered at the mean of the original set of 8 *class sizes.*

   $$\frac{19+22+23+25+26+27+30+32}{8} = 25.5 \text{ students}$$

3. **What is the largest possible range of the distribution of group mean class size?**

   The range of the distribution of mean class sizes would be the difference of the largest possible group mean and the smallest possible group mean.

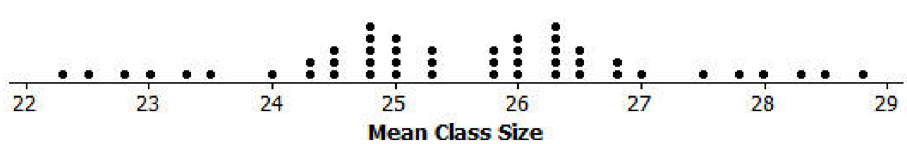
   Largest possible group mean $= \frac{26+27+30+32}{4} = 28.75.$

   Smallest possible group mean $= \frac{19+22+23+25}{4} = 22.25.$

   Largest possible group mean range $= 28.75 - 22.25 = 6.5$ *students.*

4. **What possible values for the mean class size are more likely to happen than others? Explain why you chose these values.**

   Below is a sample partial distribution of the group mean class sizes:

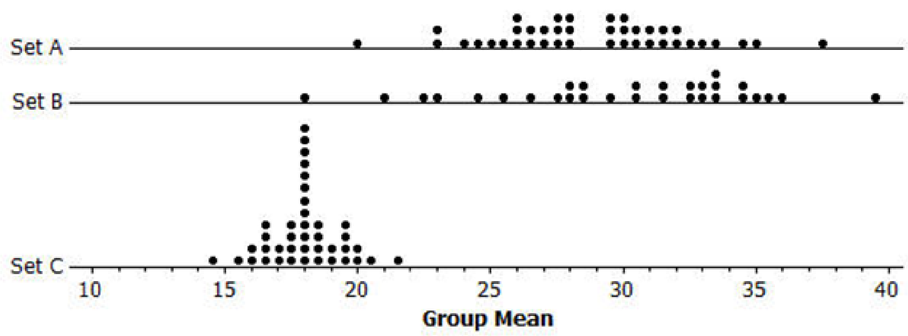


There are 3 different sets of numbers: Set A, Set B, and Set C. Each set contains 10 numbers. In two of the sets, the 10 numbers were randomly divided into two groups of 5 numbers each, and the mean for each group was calculated. These two means are plotted on a dot plot. This procedure was repeated many times, and the dot plots of the group means are shown below.

The third set did not use the above procedure to compute the means.

For each set, the smallest possible group mean and the largest possible group mean were calculated, and these two means are shown in the dot plots below.

Use the dot plots below to answer Problems 5 – 8.



5. **Which set is NOT one of the two sets that were randomly divided into two groups of 5 numbers? Explain.**

   Set B is not one of the two sets that were randomly divided into two subsets. When a set of numbers is divided into two equal groups, the resulting dot plot of group means will be symmetrical.

6. **Estimate the mean of the original values in Set A. Show your work.**

   The estimated mean of Set A is $(27 + 29.5) = 28.25$.

7. **Estimate the range of the group means shown in the dot plot for Set C. Show your work.**

   The estimated range of the distribution of sample means for Set C is $21.5 - 14.5 = 7$.

8. **Is the range of the original values in Set C smaller or larger than your answer in Problem 7? Explain.**

   The range of Set C is larger than the range of the distribution of sample means from Set C. The range of the group means will always be smaller than the range of the original set of values.

Appendix A

65	40	55	90
70	75	80	75
55	70	50	60
65	65	95	70
110	65	60	70