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Lesson 21: Margin of Error when Estimating a Population Mean

Student Outcomes

* Students use data from a random sample to estimate a population mean.
* Students calculate and interpret margin of error in context.
* Students know the relationship between sample size and margin of error in the context of estimating a population mean.

Lesson Notes

In the previous lesson, students estimated the population mean using the sample mean based on a random sample of size . To determine how accurate their estimate was, they had to create a sampling distribution of the sample mean based on computing sample means for a large number of random samples. Finally, they computed the margin of error as twice the standard deviation of the sample means. Although the process was a lot of work, students developed a conceptual understanding of margin of error.

In this lesson, students use a formula for the standard deviation of the sample mean, , where is the standard deviation of the sample and is the size of the sample. The margin of error, is based on a single random sample, thus making the work much easier.

The formula is used to calculate the standard deviation of the sample mean when the mean and the standard deviation of the population are stated. Previously, the formula was used to calculate the standard deviation of a sample *proportion* when the number of successes was known. In both formulas, as gets larger, the standard deviation gets smaller. Both methods are applications of the central limit theorem, which says that regardless of the shape of the population from which samples are taken, the distributions of both the sample means and the sample proportions are approximately normal.

Classwork

This lesson continues to discuss using the sample mean as an estimate of the population mean and judging its accuracy based on the concept of margin of error. In the last lesson, the margin of error was defined as twice the standard deviation of the sampling distribution of the sample mean. In this lesson, a formula will be given for the margin of error that allows you to calculate the margin of error from a single random sample rather than having to create a sampling distribution of sample means.

Example 1 (5 minutes): Estimating a Population Mean Using a Random Sample

Give students a few minutes to read the introductory material of this example, and remind them of the process they used in the previous lesson to get an estimate of margin of error. Then, write the formula for margin of error on the board, making sure that students understand that this will allow them to calculate an estimate of the margin of error using data from a single random sample.

**MP.2**

Example 1: Estimating a Population Mean Using a Random Sample

Provide a one-sentence summary of our findings from the previous lesson.

Sample response: We took lots of random samples of computer game ratings, computed their means, displayed the distribution of their means, and, finally, computed a margin of error.

What were drawbacks of the calculation method?

Sample response: Many samples are required. If we had increased the sample size or the number of samples, the time required to take all those samples, calculate their means, and analyze the distribution would have increased significantly.

*Scaffolding:*

* For struggling students – the bigger the value of , the smaller the standard deviation. From a population where , if

, the standard deviation is ; however, if the sample is larger, say , the standard deviation would be .

* For advanced students – when a sample is taken from a population, the mean of the sample is the same as the mean of the population, but the *variance* (square of the standard deviation) is only as large. Regardless of the shape of the population, the distribution of the sample means will approach normal (central limit theorem). The variance is ; therefore, by applying a square root, the standard deviation is .

In practice, you do not have to use that process to find the margin of error. Fortunately, just as was the case with estimating a population proportion, there are some general results that lead to a formula that allows you to estimate the margin of error using a single sample. You can then gauge the accuracy of your estimate of the population mean by calculating the margin of error using the sample standard deviation.

The standard deviation of the distribution of sample means is approximated by where is the standard deviation of the sample, and is the size of the sample.

Exercises 1–5 (10 minutes)

Have students work independently on the calculations required to answer Exercises 1–3. Then, work through Exercises 4 and 5 as a class.

Exercises 1–5

1. Suppose a random sample of size ten produced the following ratings in the computer games rating example in the last lesson: . Estimate the population mean rating based on these ten sampled ratings.

The sample mean estimate for the population mean rating is rating points.

1. Calculate the sample standard deviation. Round your answer to three decimal places.

The sample standard deviation is rating points.

1. Use the formula given above to calculate the approximate standard deviation of the distribution of sample means. Round your answer to three decimal places.

The standard deviation of the distribution of sample means is rating points.

1. Recall that the margin of error is twice the standard deviation of the distribution of sample means. What is the value of the margin of error based on this sample? Write a sentence interpreting the value of the margin of error in the context of this problem on computer game ratings.

Margin of error is rating points. The population mean rating for the computer games is likely to be within rating points of the sample mean estimate .

1. Based on the sample mean and the value of the margin of error, what is an interval of plausible values for the population mean?

Plausible values for the population mean rating are from to rating points.

**Exercises 6–13 (20 minutes): The Gettysburg Address**

Distribute a copy of the Gettysburg Address to each student in the class. (A copy is provided at the end of this lesson.) Have students work individually or in pairs to answer the questions in this set of exercises. Then, discuss the answers to the last question as a class. Consider challenging students to find the length of a typical word in the Gettysburg Address.

After students do this exercise “by hand” (using a calculator), you may want to show them an applet that displays three different estimates regarding the Gettysburg Address. One is the mean word length. The other two are estimating population proportions; one is the proportion of “long” words defined as words with more than four letters, and the other is the proportion of nouns. The applet can be found at the following site: <http://www.rossmanchance.com/applets/GettysburgSample/GettysburgSample.html>.

This applet may require an updated version of an operating system to work correctly. If the applet does not work for all students due to a computer’s operating system or network settings, attempt to demonstrate it for the whole class, as it is an effective way to complement how students obtained their answers in the exercises. The applet allows the user to specify a sample size (ten in this exercise) and the number of samples desired. Note that only *one* sample is to be used to answer the questions in this exercise set.

To generate a sampling distribution for the sample mean (or proportion), enter a large number in the Num samples box, such as . The Animate box shows the observations for each sample taken and the resulting values of the statistics (mean or proportion) plotted on a histogram. (You may unclick the Animate box at any time to see the total results immediately.)

Students should begin work on Exercise 6. Exercises 7–13 are provided as scaffolding if necessary. Students should be able to clearly describe and fully implement a plan on their own. Sample responses are provided but will vary.

Exercises 6-13

The Gettysburg Address is considered one of history’s greatest speeches. Some students noticed that the speech was very short (about words, depending on the version) and wondered if the words were also relatively short. To estimate the mean length of words in the population of words in the Gettysburg Address, work with a partner on the following steps. Your teacher will give you a copy of the Gettysburg Address with words numbered from to .

1. Develop and describe a plan for collecting data from the Gettysburg Address and determining the typical length of a word. Then, implement your plan, and report your findings.

Many answers are possible. Every answer should include the following:

* A description of how a word sample is chosen, making sure to describe how randomization occurs
* The actual sample chosen
* Calculations of the sample mean, standard deviation, and margin of error
* Interpretations in context of the sample mean, standard deviation, and margin of error
1. Use a random-number table or a calculator with a random-number generator to obtain ten different random numbers from to .

**MP.5**

1. Use the random numbers found in Exercise 7 as identification numbers for the words that will make up your random sample of words from the Gettysburg Address. Make a list of the ten words in your sample.

|  |  |
| --- | --- |
|  the resolve score consecrate final |  The nobly a The endure |

1. Count the number of letters in each of the ten words in your sample.
2. Calculate the sample mean number of letters for the ten words in your sample.

The mean of the ten word lengths from Exercise 9 is letters.

1. Calculate the sample standard deviation of the number of letters for the ten words in your sample. Round your answer to three decimal places.

The standard deviation of the ten word lengths from Exercise 9 is letters.

1. Use the sample standard deviation from Exercise 11 to calculate the margin of error associated with using your sample mean as an estimate of the population mean. Round your answer to three decimal places.

The margin of error of this estimate is = letters.

**MP.2**

1. Write a few sentences describing what you have learned about the mean length of the population of words in the Gettysburg Address. Be sure to include an interpretation of the margin of error.

We estimate the mean word length of words in the Gettysburg Address to be letters. The margin of error of this estimate is letters. So, plausible values for the population mean word length are from to letters.

Closing (3 minutes)

* Ask students to summarize the main ideas of the lesson in writing or with a neighbor. Use this as an opportunity to informally assess comprehension of the lesson. The Lesson Summary below offers some important ideas that should be included.

Lesson Summary

* When using the sample mean to estimate a population mean, it is important to know something about how accurate that estimate might be.
* Accuracy can be described by the margin of error.
* The margin of error can be estimated using data from a single random sample (without the need to create a simulated sampling distribution) by using the formula where is the standard deviation of a single sample, and is the sample size.

Exit Ticket (7 minutes)

Name Date

Lesson 21: Margin of Error when Estimating a Population Mean

Exit Ticket

A Health Group study recommends that the total weight of a male student’s backpack should not be more that of his body weight. For example, if a student weighs pounds, his backpack should not weigh more than pounds. Suppose that ten randomly selected eleventh grade boys produced the following data:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Body weight |  |  |  |  |  |  |  |  |  |  |
| Backpack weight |  |  |  |  |  |  |  |  |  |  |

* 1. For each student, calculate backpack weight as a percentage of body weight (round to one decimal place).
	2. Based on the data in part (a), estimate the mean percentage of body weight that eleventh grade boys carry in their backpacks.
	3. Find the margin of error for your estimate of part (b). Round your answer to three decimal places. Explain how you determined your answer.
	4. Comment on the amount of weight eleventh grade boys at this school are carrying in their backpacks compared to the recommendation by the Health Group.

Exit Ticket Sample Solutions

A Health Group study recommends that the total weight of a male student’s backpack should not be more that of his body weight. For example, if a student weighs pounds, his backpack should not weigh more than pounds. Suppose that ten randomly selected eleventh grade boys produced the following data:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Body weight |  |  |  |  |  |  |  |  |  |  |
| Backpack weight |  |  |  |  |  |  |  |  |  |  |

* 1. For each student, calculate backpack weight as a percentage of body weight (round to one decimal place).

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Body weight |  |  |  |  |  |  |  |  |  |  |
| Backpack weight |  |  |  |  |  |  |  |  |  |  |
| Percentage |  |  |  |  |  |  |  |  |  |  |

* 1. Based on the data in part (a), estimate the mean percentage of body weight that eleventh grade boys carry in their backpacks.

The sample mean percentage is percentage points.

* 1. Find the margin of error for your estimate of part (b). Explain how you determined your answer.

The standard deviation of the percentages ispercent. So, the margin of error is percentage points.

* 1. Comment on the amount of weight eleventh grade boys at this school are carrying in their backpacks compared to the recommendation by the Health Group.

Based on the data in this study, plausible percentages of mean body weight percentage that eleventh graders are carrying in their backpacks are to percentage points. The interval is above the recommended maximum. On average, eleventh grade boys at this school are carrying too much weight in their backpacks.

Problem Set Sample Solutions

1. A new brand of hot dog claims to have a lower sodium content than the leading brand.
	1. A random sample of ten of these new hot dogs results in the following sodium measurements (mg).

Estimate the population mean sodium content of this new brand of hot dog based on the ten sampled measurements.

Based on the data, an estimate for the population mean sodium content of this new brand of hot dog is mg of sodium.

* 1. Calculate the margin of error associated with your estimate of the population mean from part (a). Round your answer to three decimal places.

The margin of error is mg.

* 1. The mean sodium content of the leading brand of hot dogs is known to be mg. Based on the sample mean and the value of the margin of error for the new brand, is a mean sodium content of mg a plausible value for the mean sodium content of the new brand? Comment on whether you think the new brand of hot dog has a lower sodium content on average than the leading brand.

Plausible values for population mean sodium content are between mg and mg. This interval is well below the mg which is the mg content for the leading brand. So, the new hot dog brand has lower mean sodium content.

* 1. Another random sample of new brand hot dogs is taken. Should this larger sample of hot dogs produce a more accurate estimate of the population mean sodium content than the sample of size ? Explain your answer by appealing to the formula for margin of error.

The margin of error will be smaller. Sample size is in the denominator of formula for margin of error.

1. It is well known that astronauts increase their height in space missions because of the lack of gravity. A question is whether or not we increase height here on Earth when we are put into a situation where the effect of gravity is minimized. In particular, do people grow taller when confined to a bed? A study was done in which the heights of six men were taken before and after they were confined to bed for three full days.
	1. The before-after differences in height measurements (mm) for the six men were:

Assuming that the men in this study are representative of the population of all men, what is an estimate of the population mean increase in height after three full days in bed?

Based on the given data, an estimate of the population mean increase in height after three full days in bed is mm.

* 1. Calculate the margin of error associated with your estimate of the population mean from part (a). Round your answer to three decimal places.

The margin of error is approximated by mm.

* 1. Based on your sample mean and the margin of error from parts (a) and (b), what are plausible values for the population mean height increase for all men who stay in bed for three full days?

Plausible values for the population mean height increase for all men who stay in bed for three full days are those between and mm.

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| 001 Four | 045 any | 089 nation | 133 our | 177 they | 221 full | 265 perish |
| 002 score | 046 nation, | 090 might | 134 poor | 178 who | 222 measure | 266 from |
| 003 and | 047 so | 091 live. | 135 power | 179 fought | 223 of | 267 the |
| 004 seven | 048 conceived | 092 It | 136 to | 180 here | 224 devotion, | 268 earth. |
| 005 years | 049 and | 093 is | 137 add | 181 have | 225 that |  |
| 006 ago, | 050 so | 094 altogether | 138 or | 182 thus | 226 we |  |
| 007 our | 051 dedicated, | 095 fitting | 139 detract. | 183 far | 227 here |  |
| 008 fathers | 052 can | 096 and  | 140 The | 184 so | 228 highly |  |
| 009 brought | 053 long | 097 proper  | 141 world | 185 nobly  | 229 resolve |  |
| 010 forth | 054 endure. | 098 that | 142 will | 186 advanced. | 230 that |  |
| 011 upon | 055 We | 099 we | 143 little | 187 It | 231 these |  |
| 012 this | 056 are | 100 should  | 144 note, | 188 is | 232 dead |  |
| 013 continent | 057 met | 101 do | 145 nor | 189 rather | 233 shall |  |
| 014 a | 058 on | 102 this. | 146 long | 190 for | 234 not |  |
| 015 new | 059 a  | 103 But, | 147 remember, | 191 us | 235 have |  |
| 016 nation; | 060 great | 104 in  | 148 what | 192 to | 236 died |  |
| 017 conceived | 061 battlefield | 105 a | 149 we | 193 be | 237 in |  |
| 018 in | 062 of | 106 larger | 150 say | 194 here | 238 vain, |  |
| 019 liberty, | 063 that | 107 sense, | 151 here, | 195 dedicated | 239 that |  |
| 020 and | 064 war. | 108 we | 152 but | 196 to  | 240 this |  |
| 021 dedicated | 065 We  | 109 cannot | 153 it | 197 the | 241 nation,  |  |
| 022 to | 066 have | 110 dedicate, | 154 can  | 198 great | 242 under  |  |
| 023 the | 067 come | 111 we | 155 never | 199 task | 243 God,  |  |
| 024 proposition | 068 to  | 112 cannot | 156 forget | 200 remaining | 244 shall |  |
| 025 that | 069 dedicate | 113 consecrate, | 157 what | 201 before | 245 have |  |
| 026 all | 070 a | 114 we | 158 they | 202 us, | 246 a |  |
| 027 men | 071 portion | 115 cannot | 159 did | 203 that | 247 new |  |
| 028 are | 072 of | 116 hallow | 160 here. | 204 from | 248 birth |  |
| 029 created | 073 that | 117 this | 161 It | 205 these | 249 of  |  |
| 030 equal. | 074 field | 118 ground. | 162 is | 206 honored | 250 freedom, |  |
| 031 Now | 075 as | 119 The | 163 for | 207 dead | 251 and |  |
| 032 we | 076 a | 120 brave | 164 us | 208 we | 252 that |  |
| 033 are | 077 final | 121 men, | 165 the | 209 take | 253 government |  |
| 034 engaged | 078 resting | 122 living | 166 living, | 210 increased | 254 of |  |
| 035 in | 079 place | 123 and | 167 rather, | 211 devotion | 255 the |  |
| 036 a | 080 for  | 124 dead, | 168 to | 212 to  | 256 people, |  |
| 037 great | 081 those | 125 who | 169 be | 213 that | 257 by |  |
| 038 civil | 082 who  | 126 struggled | 170 dedicated | 214 cause | 258 the |  |
| 039 war, | 083 here | 127 here | 171 here | 215 for  | 259 people, |  |
| 040 testing | 084 gave | 128 have | 172 to | 216 which | 260 for |  |
| 041 whether | 085 their | 129 consecrated | 173 the | 217 they | 261 the |  |
| 042 that | 086 lives | 130 it,  | 174 unfinished | 218 gave | 262 people, |  |
| 043 nation, | 087 that | 131 far | 175 work | 219 the | 263 shall |  |
| 044 or | 088 that | 132 above | 176 which | 220 last | 264 not |  |

Exercises 6-13: Gettysburg Address