Lesson 19: Sampling Variability in the Sample Mean

Student Outcomes

- Students understand the term "sampling variability" in the context of estimating a population mean.
- Students understand that the standard deviation of the sampling distribution of the sample mean conveys information about the anticipated accuracy of the sample mean as an estimate of the population mean.

Lesson Notes

This is the second of two lessons building on the concept of sampling variability in the sample mean first developed in Grade 7 (Module 5, Lessons 17–19). Students use simulation to approximate the sampling distribution of the sample mean for random samples from a population. They also explore how the simulated sampling distribution provides information about the anticipated estimation error when using a sample mean to estimate a population mean and how sample size affects the distribution of the sample mean.

Classwork

This lesson uses simulation to approximate the sampling distribution of the sample mean for random samples from a population, explores how the simulated sampling distribution provides insight into the anticipated estimation error when using a sample mean to estimate a population mean, and covers how sample size affects the distribution of the sample mean.

Exercises 1–6 (35 minutes): SAT scores

In this lesson, you may want to give students the population data and have them use technology to take random samples (without replacement) from the population. (You could copy and paste the table into a spreadsheet and send it to students.) A typical set of commands to generate a random sample might be: randsamp(SAT_scores, 20), where SAT_scores is the name of the list containing the scores and 20 is the sample size. The random sample should refresh by clicking on the command line or by using a command such as Control R.

Note that the sample answers for the simulated distributions typically display the means from about 50 random samples. If it is possible to generate many more samples quickly with technology, students should do so. The basic characteristics of a distribution of sample means (center, spread, mound shaped) do not change much as more samples are added to the first 50 or so—which is suggested by contrasting Exercise 3 parts (a) and (b)—and students should experience this themselves by generating their own distributions with many samples.

Part (a) of both Exercises 2 and 3 are important to discuss as they highlight the difference in the distribution of the values in the sample (the scores) and the distribution of the sample means (the mean of the scores) in a sample.

Have students share the simulated sampling distributions they generate for Exercise 3. You might use screen capture/sharing software or have students walk around the room with post-it notes looking at each student's handheld or computer screen and recording what they see about the distributions. Recognizing that all of the simulated distributions have essentially the same characteristics is a key factor in understanding why it is possible to make general statements about how random samples behave in relation to the population.

MP.2

Exercises 1–6: SAT scores

1. **SAT test scores vary a lot. The table displays the 506 scores for students in one New York school district for a given year.**

Table 1: SAT scores for district students

441	395	369	350	521	691	648	521	498	413	486
440	415	481	392	800	448	603	503	486	476	500
391	359	447	550	432	158	379	394	495	442	507
395	504	399	424	456	729	356	392	514	388	518
445	436	386	493	467	493	440	387	512	431	467
499	412	457	389	323	319	550	450	517	405	506
486	519	369	373	348	532	496	488	504	444	
396	473	319	367	679	472	613	561	522	408	
451	427	369	560	602	520	567	495	473	424	
362	391	371	407	436	366	582	528	533	463	
328	613	357	438	436	713	603	525	553	446	
414	466	382	362	777	259	557	508	495	466	
409	486	627	589	749	410	639	516	520	632	
526	334	608	374	634	443	556	506	506	526	
391	497	378	358	566	442	496	568	544	546	
529	392	387	373	198	555	499	476	525	529	
529	426	470	378	345	431	613	490	548	455	
574	379	380	561	712	197	556	547	543	431	
363	382	370	379	504	254	596	489	474	386	
486	434	365	530	685	372	580	506	529	434	
418	722	674	504	645	501	605	511	566	362	
527	437	388	525	509	662	445	489	487	426	
441	395	377	561	448	503	602	523	510	404	
467	463	427	519	491	448	638	530	518	493	
387	433	446	525	352	662	570	507	515	515	
503	371	394	569	779	158	558	504	516	407	
350	392	368	484	689	691	535	522	505	409	
583	416	406	416	513	729	623	503	536	422	
370	370	350	446	624	493	465	524	547	612	
499	422	344	420	465	319	460	523	528	486	
399	532	347	446	504	532	375	524	527	394	
374	545	377	462	390	472	540	501	523	424	
372	427	391	528	576	520	564	482	540	393	
559	371	339	533	756	366	547	502	480	420	
330	390	404	543	451	713	568	503	516	415	
567	529	377	460	505	259	588	439	501	394	
371	341	469	391	540	410	502	474	452	473	
503	356	417	623	436	443	510	477	507	531	
327	351	356	587	298	442	589	458	486	469	
528	377	370	528	449	555	537	494	500	453	
447	404	355	356	352	431	410	447	507	442	
572	369	364	523	574	197	330	517	518	509	
379	396	383	404	518	460	500	457	467	435	
456	396	400	505	682	623	531	471	506	427	
406	535	404	512	474	587	509	541	509	489	
420	388	375	514	629	528	571	513	597	480	
395	370	398	516	656	523	527	441	509	516	
355	417	376	498	539	505	457	489	567	501	
423	419	451	460	553	514	552	498	509	452	
438	348	369	541	400	629	561	538	597	507	

a. Looking at the table above, how would you describe the population of SAT scores?

Sample response: It is hard to tell. I see some numbers as low as 198 and others in the 700s. You cannot tell much from just looking at the individual numbers.

b. Jason used technology to draw a random sample of size 20 from all of the scores and found a sample mean of 487. What does this value represent in terms of the graph below?

Random sample from District SAT scores

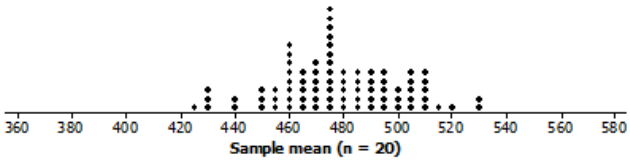
Sample response: This represents the average SAT score for the sample and indicates where the scores in the dot plot are centered. If you computed the mean of the values for the SAT scores in the sample (i.e., one student had an SAT score about 350, two had scores a bit over 350, one student scored about 625, and so on), you would find the mean SAT scores for those students to be 487.

2. If you were to take many different random samples of 20 from this population, describe what you think the sampling distribution of these sample means would look like.

Sample response: Maybe centered on a value close to 487. I am not sure of the spread though, possibly from 350 to 600 like the sample in the example.

3. Everyone in Jason's class drew several random samples of size 20 and found the mean SAT score. The plot below displays the distribution of the mean SAT scores for their samples.

Random sample from District SAT scores



a. How does the simulated sampling distribution compare to your conjecture in Exercise 2? Explain any differences.

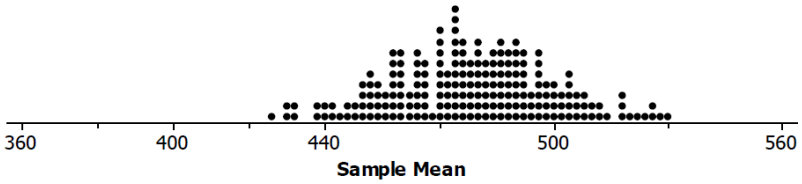
Possible answer: I estimated the mean SAT score to be a bit higher than it seems to be from the simulated distribution, but my estimate was not that far off. I thought the spread would range from 350 to 600 like in Jason's sample, but that was not a good estimate. I was thinking of the individual SAT scores in the sample and mixing them up with the means of samples of 20 scores, which is why my estimate was off.

MP.5

b. Use technology to generate many more samples of size 20, and plot the means of those samples. Describe the shape of the simulated distribution of sample mean SAT scores.

Answers will vary. The following displays an example of a simulated distribution of sample means from random samples from District SAT scores

The distribution is normal. The mean of the sample mean SAT scores appears to be around 480, and the sample mean scores range from a little over 420 to about 530.



c. How did the simulated distribution using more samples compare to the one you generated in Exercise 3?

Sample response: The maxima and minima were nearly the same in both of the distributions, 420 to 530 and a little over 420 to 520. The mean SAT scores of the simulated distributions of sample means in both seemed to be about 480.

d. What are the mean and standard deviation of the simulated distribution of the sample mean SAT scores you found in part (b)? (Use technology and your simulated distribution of the sample means to find the values.)

Sample response: The mean SAT score of the simulated distribution of sample means is 478. The standard deviation is 21.5.

e. Write a sentence describing the distribution of sample means that uses the mean and standard deviation you calculated in part (d).

Sample response: Almost all of the SAT scores are within two standard deviations from the mean, from 435 to 521.

4. Reflect on some of the simulated sampling distributions you have considered in previous lessons.

a. Make a conjecture about how you think the size of the sample might affect the distribution of the sample SAT means.

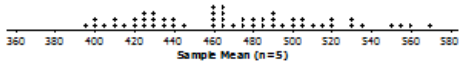
Sample response: the larger the sample size, the smaller the spread of the distribution of sample means.

MP.5

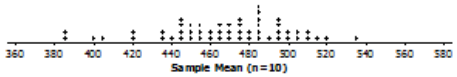
b. To test the conjecture, investigate the following sample sizes: $5, 10, 40$, and 50 as well as the simulated distribution of sample means from Exercise 3. Divide the sample sizes among your group members, and use technology to simulate sampling distributions of mean SAT scores for samples of the different sizes. Find the mean and standard deviation of each simulated sampling distribution.

The following represent simulated distributions of sample means of District SAT scores for different size random samples:

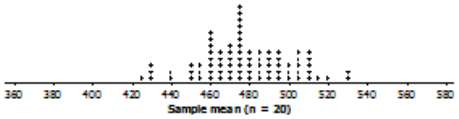
mean 469; standard deviation is 45.4



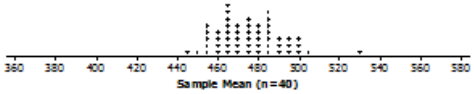
mean 468; standard deviation is 32.9



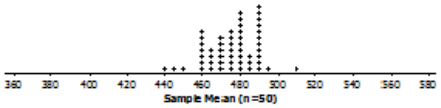
mean 478; standard deviation is 22.8



mean 475; standard deviation is 16.1



mean 476; standard deviation is 13.8



c. **How does the sample size seem to affect the simulated distributions of the sample SAT mean scores? Include the simulated distribution from part (b) of Exercise 3 in your response. Why do you think this is true?**

Sample response: As the sample size increases, the spread decreases. The standard deviation went from 40 for a sample of size 5 to about 14 for a sample of size 50. The means of the sampling distributions of mean SAT scores varied from 468 for the distribution for samples of size 10 to 478 for samples of size 20. I would expect that a bigger sample would be more likely to look a lot like the population, and so bigger samples wouldn't tend to be as different from one another as smaller samples. Because of this, the sample means wouldn't differ as much from sample to sample for bigger samples.

5.

a. **For each of the sample sizes, consider how the standard deviation seems to be related to the range of the sample means in the simulated distributions of the sample SAT means you found in Exercise 4.**

Sample response: In each case, nearly all of the sample means are within two standard deviations of the mean or are a normal distribution.

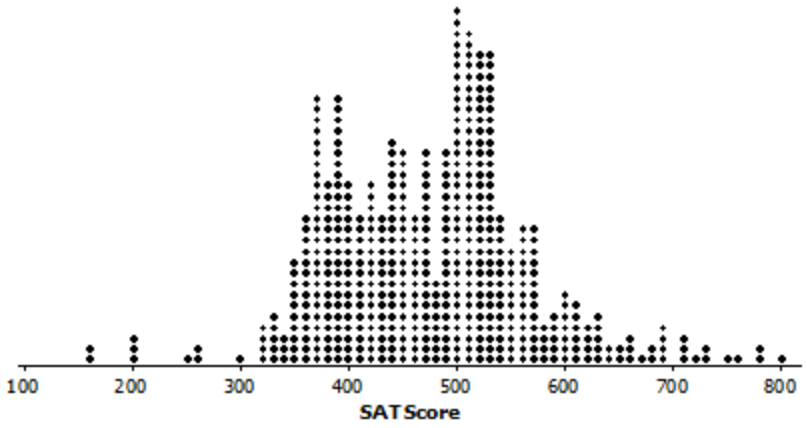
b. **How do your answers to part (a) compare to the answers from other groups?**

Possible answer: Everyone had simulated sampling distributions that looked fairly alike and were centered in about the same place. The sample means for each sampling distribution were typically within two standard deviations of the mean of the simulated sampling distributions. The distributions were normal.

6.

a. **Make a graph of the distribution of the population consisting of the SAT scores for all of the students.**

Possible response below of a distribution of the SAT scores for district students



b. **Find the mean of the distribution of SAT scores. How does it compare to the mean of the sampling distributions you have been simulating?**

Possible answer: The mean SAT score for the students in the district is 475.1 or 475. This is close to the means of the sampling distributions, even for fairly small samples.

Closing (5minutes)

- Why do we call the sampling distributions we generated "*simulated* sampling distribution of sample means" rather than the "sampling distribution of sample means"?
  - *The sampling distribution of sample means is the distribution of all the possible sample means for a sample of a given size (i.e., every possible combination of the population values for that size). The simulated sampling distribution is a subset of the actual sampling distribution that, because it is random, approximates the actual sampling distribution.*

- If you had a simulated distribution of the mean SAT scores for random samples of size 100, how do you think the distribution would compare to the distribution you found for samples of size 50?
  - *Sample response: The mean would be somewhere around 475, and the standard deviation would be smaller, so the values (the sample means) would be closer together.*
- Ask students to summarize the main ideas of the lesson in writing or with a neighbor. Use this opportunity to informally assess comprehension of the lesson. The Lesson Summary below offers some important ideas that should be included.

Lesson Summary

For a given sample you can find the sample mean.

- **There is variability in the sample mean. The value of the sample mean varies from one random sample to another.**
- **A graph of the distribution of sample means from many different random samples is a simulated sampling distribution.**
- **Sample means from random samples tend to cluster around the value of the population mean. That is, the simulated sampling distribution of the sample mean will be centered close to the value of the population mean.**
- **The variability in the sample mean decreases as the sample size increases.**
- **Most sample means are within two standard deviations of the mean of the simulated sampling distribution.**

Exit Ticket (5 minutes)

Name _____ Date _____

Lesson 19: Sampling Variability in the Sample Mean

Exit Ticket

1. Describe the difference between a population distribution, a sample distribution, and a simulated sampling distribution and make clear how they are different.

2. Use the standard deviation and mean of the sampling distribution to describe an interval that includes most of the sample means.

Exit Ticket Sample Solutions

1. **Describe the difference between a population distribution, a sample distribution, and a simulated sampling distribution and make clear how they are different.**

   Possible answer: The distribution of the elements in a population (the SAT scores for students in a district) is a population distribution; the distribution of the elements in a random sample from that population (a subset of a given size chosen at random from the SAT scores) is a sample distribution; a simulated distribution of sample means for many random samples of a given size chosen from the population (the means of different random samples of the same size of students' SAT scores) is a simulated sampling distribution.

   Some students might also suggest that the meaning of sampling distribution of all samples is the samples of a given size selected from a population. This would be the distribution of the means of every possible sample that might be chosen.

2. **Use the standard deviation and mean of the sampling distribution to describe an interval that includes most of the sample means.**

   Sample response: Typically, most of the means of the different random samples of the same size chosen from a population will be within two standard deviations of the mean or the Mean $+/-$ 2 standard deviations.

Problem Set Sample Solutions

1. **Which of the following will have the smallest standard deviation? Explain your reasoning.**

   A sampling distribution of sample means for samples of size:

   a. 15 **b.** 25 **c.** 100

   Possible answer: The largest sample size, 100, will have the smallest standard deviation because as the sample size increases, the variability in the sample mean decreases.

2. **In light of the distributions of sample means you have investigated in the lesson, comment on the statements below for random samples of size 20 chosen from the District SAT scores.**

   a. **Josh claimed he took a random sample of size 20 and had a sample mean score of 320.**

      Possible answer: A mean score of 320 seems very unlikely. None of the samples we have investigated had a sample mean score that low.

   b. **Sarfina stated she took a random sample of size 20 and had a sample mean of 520.**

      Possible answer: This seems plausible for the simulated distributions of sample mean scores; 520 was high but still some of the random samples had mean scores greater than 520.

   c. **Ana announced that it would be pretty rare for the mean SAT score in a random sample to be more than three standard deviations from the mean SAT score of 475.**

      Possible answer: Given that the sample means in nearly all of the simulated distributions of the sample means were usually within two standard deviations from the mean, Ana is correct. It could happen, but it would not be usual.

3. Refer to your answers for Exercise 4, and then comment on each of the following:

   a. A random sample of size 50 produced a mean SAT score of 400.

   Sample response: A mean score of 400 was less than any of the sample means in the simulated sampling distribution of sample means for samples of size 50, so this seems unlikely.

   b. A random sample of size 10 produced a mean SAT score of 400.

   Sample response: A mean score of 400 was within two standard deviations of the mean for random samples of size 10, so it could have come from one of the samples.

   c. For what sample sizes was a sample mean SAT score of 420 plausible? Explain your thinking.

   Sample response: A sample mean of 420 occurred in the simulated sampling distributions for samples of size 5 and 10 but not at all in the simulated distributions for samples of size $20, 40$ and 50. So it seems like 420 was a plausible outcome for samples of size 5 and 10.

4. Explain the difference between the sample mean and the mean of the sampling distribution.

   Possible answer: Each sample of SAT scores had a mean SAT score, which is the sample mean. Then all of those sample means formed a distribution of sample means, and we found the mean of that set, the mean of the sampling distribution of the sample mean – the mean of the means of the different samples.