Lesson 19: Sampling Variability in the Sample Mean

Classwork

This lesson uses simulation to approximate the sampling distribution of the sample mean for random samples from a population, explores how the simulated sampling distribution provides insight into the anticipated estimation error when using a sample mean to estimate a population mean, and covers how sample size affects the distribution of the sample mean.

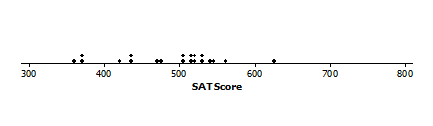
Exercises 1–6: SAT scores

1. SAT test scores vary a lot. The table displays the scores for students in one New York school district for a given year.

**Table 1: SAT scores for district students**

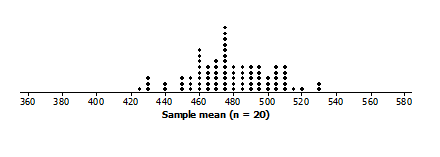
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* 1. Looking at the table above, how would you describe the population of SAT scores?
  2. Jason used technology to draw a random sample of size from all of the scores and found a sample mean of . What does this value represent in terms of the graph below?

**Random sample from District SAT scores**

1. If you were to take many different random samples of from this population, describe what you think the sampling distribution of these sample means would look like.
2. Everyone in Jason’s class drew several random samples of size and found the mean SAT score. The plot below displays the distribution of the mean SAT scores for their samples.

**Random sample from District SAT scores**



* 1. How does the simulated sampling distribution compare to your conjecture in Exercise 2? Explain any differences.
  2. Use technology to generate many more samples of size and plot the means of those samples. Describe the shape of the simulated distribution of sample mean SAT scores.
  3. How did the simulated distribution using more samples compare to the one you generated in Exercise 3?
  4. What are the mean and standard deviation of the simulated distribution of the sample mean SAT scores you found in part (b)? (Use technology and your simulated distribution of the sample means to find the values.)
  5. Write a sentence describing the distribution of sample means that uses the mean and standard deviation you calculated in part (d).

1. Reflect on some of the simulated sampling distributions you have considered in previous lessons.
   1. Make a conjecture about how you think the size of the sample might affect the distribution of the sample SAT means.
   2. To test the conjecture, investigate the following sample sizes: and as well as the simulated distribution of sample means from Exercise 3. Divide the sample sizes among your group members, and use technology to simulate sampling distributions of mean SAT scores for samples of the different sizes. Find the mean and standard deviation of each simulated sampling distribution.
   3. How does the sample size seem to affect the simulated distributions of the sample SAT mean scores? Include the simulated distribution from part (b) of Exercise 3 in your response. Why do you think this is true?
   4. For each of the sample sizes, consider how the standard deviation seems to be related to the range of the sample means in the simulated distributions of the sample SAT means you found in Exercise 4.
   5. How do your answers to part (a) compare to the answers from other groups?
   6. Make a graph of the distribution of the population consisting of the SAT scores for all of the students.



* 1. Find the mean of the distribution of SAT scores. How does it compare to the mean of the sampling distributions you have been simulating?

Problem Set

Lesson Summary

For a given sample you can find the sample mean.

* There is variability in the sample mean. The value of the sample mean varies from one random sample to another.
* A graph of the distribution of sample means from many different random samples is a simulated sampling distribution.
* Sample means from random samples tend to cluster around the value of the population mean. That is, the simulated sampling distribution of the sample mean will be centered close to the value of the population mean.
* The variability in the sample mean decreases as the sample size increases.
* Most sample means are within two standard deviations of the mean of the simulated sampling distribution.

1. Which of the following will have the smallest standard deviation? Explain your reasoning.

A sampling distribution of sample means for samples of size:

a. b. c.

1. In light of the distributions of sample means you have investigated in the lesson, comment on the statements below for random samples of size chosen from the District SAT scores.
   1. Josh claimed he took a random sample of size and had a sample mean score of .
   2. Sarfina stated she took a random sample of size and had a sample mean of .
   3. Ana announced that it would be pretty rare for the mean SAT score in a random sample to be more than three standard deviations from the mean SAT score of .
2. Refer to your answers for Exercise 4, and then comment on each of the following:
   1. A random sample of size produced a mean SAT score of .
   2. A random sample of size produced a mean SAT score of .
   3. For what sample sizes was a sample mean SAT score of plausible? Explain your thinking.
3. Explain the difference between the sample mean and the mean of the sampling distribution.