Lesson 16: Margin of Error when Estimating a Population Proportion

Classwork

Exercises 1-4

In this lesson you will use data from a random sample drawn from a mystery bag to estimate a population proportion and learn how to find and interpret a margin of error for your estimate.

1. Write down your estimate for the proportion of red chips in the mystery bag based on the random sample of $30$ chips drawn in class.
2. Tanya and Raoul had a paper bag that contained red and black chips. The bag was marked $40\%$ red chips. They drew random samples of $30$ chips, with replacement, from the bag. (They were careful to shake the bag after they replaced a chip.) They had nine red chips in their sample. They drew another random sample of $30$ chips from the bag, and this time they had $12$ red chips. They repeated this sampling process $50$ times and made a plot of the number of red chips in each sample. A plot of their sampling distribution is shown below.



* 1. What was the most common number of red chips in the $50$ samples? Does this seem reasonable? Why or why not?
	2. What number of red chips, if any, never occurred in any of the samples?
	3. Give an interval that contains the “likely” number of red chips in samples of size $30$ based on the simulated sampling distribution.
	4. Do you think the number of red chips in the mystery bag could have come from a sample drawn from a bag that had $40\%$ red chips? Why or why not?

Nine different bags of chips were distributed to small teams of students in the class. Each bag had a different proportion of red chips. Each team simulated drawing $50$ different random samples of size $30$ from their bag and recorded the number of red chips for each sample. The graphs of their simulated sampling distributions are shown below.

|  |  |
| --- | --- |
| Population $10\%$ red chips | Population $20\%$ red chips |

|  |  |
| --- | --- |
| Population $30\%$ red chips | Population $40\%$ red chips |

|  |  |
| --- | --- |
| Population $50\%$ red chips | Population $60\%$ red chips |

|  |  |
| --- | --- |
| Population $70\%$ red chips | Population $80\%$ red chips |

|  |  |
| --- | --- |
| Population $90\%$ red chips |  |

1. Think about the number of red chips in the random sample of size $30$ that was drawn from the mystery bag.
	1. Based on the simulated sampling distributions, do you think that the mystery bag might have had $10\%$ red chips? Explain your reasoning.
	2. Based on the simulated sampling distributions, which of the percentages $10\%$, $20\%$, $30\%$, $40\%$, $50\%$, $60\%$, $70\%$, $80\%$, and $90\%$ might reasonably be the percentage of red chips in the mystery bag?
	3. Let $p$ represent the proportion of red chips in the mystery bag. (For example, $p = 0.40$ if there are $40\%$ red chips in the bag.) Based on your answer to part (b), write an inequality that describes plausible values for $p$. Interpret the inequality in terms of the mystery bag population.
2. If the inequality like the one you described in part (c) of Exercise 3 went from $0.30$ to $0.60$, it is sometimes written as $0.45 \pm 0.15$. The value $0.15$ is called a “margin of error.” The margin of error represents an interval from the expected proportion that would not contain any proportions or very few proportions based on the simulated sampling distribution. Proportions in this interval are not expected to occur when taking a sample from the mystery bag.
	1. Write the inequality you found in Exercise 3 part (c) using this notation. What is the margin of error?
	2. Suppose Sol said, “So this means that the actual proportion of red chips in the mystery bag was $60\%$.” Tonya argued that the actual proportion of red chips in the mystery bag was $20\%$. What would you say?

Exploratory Challenge 2: Samples of Size 50/Exercises 5–7

1. Do you think the “margin of error” would be different in Exercise 4 if you had sampled $50$ chips instead of $30$? Try to convince a partner that your conjecture is correct.
2. Below are simulated sampling distributions of the number of red chips for samples of size $50$ from populations with various percentages of red chips.

|  |  |
| --- | --- |
| Population with $10\%$ red chips | Population with $20\%$ red chips |

|  |  |
| --- | --- |
| Population with $30\%$ red chips | Population with $40\%$ red chips |

|  |  |
| --- | --- |
| Population with $50\%$ red chips | Population with $60\%$ red chips |

|  |  |
| --- | --- |
| Population with $70\%$ red chips | Population with $80\%$ red chips |

|  |  |
| --- | --- |
| Population with $90\%$ red chips |  |

* 1. Suppose you drew $30$ red chips in a random sample of $50$ from the mystery bag. What are plausible values for the proportion of red chips in the mystery bag? Explain your reasoning.
	2. Write an expression that contains the margin of error based on your answer to part (a).
1. Remember your conjecture from Exercise 5, and compare the margin of error you found for a sample of size $30$ (from Exercise 3) to the margin of error you found for a sample of size $50$.
	1. Was your reasoning in Exercise 5 correct? Why or why not?
	2. Explain why the change in the margin of error makes sense.

Problem Set

Lesson Summary

In this lesson, you investigated how to make an inference about an unknown population proportion based on a random sample from that population.

* You learned how random samples from populations with known proportions of successes behave by simulating sampling distributions for samples drawn from those populations.
* Comparing an observed proportion of successes from a random sample drawn from a population with an unknown proportion of successes to these sampling distributions gives you some information about what populations might produce a random sample like the one you observed.
* These plausible population proportions can be described as $p\pm M$. The value of $M$ is called a margin of error.
1. Tanya simulated drawing a sample of size $30$ from a population of chips and got the following simulated sampling distribution for the number of red chips:



Which of the following results seem like they might have come from this population? Explain your reasoning.

1. $8$ red chips in a random sample of size $30.$
2. $12$ red chips in a random sample of size $30.$
3. $24$ red chips in a random sample of size $30.$
4. $64\%$ percent of the students in a random sample of $100$ high school students intended to go onto college. The graphs below show the result of simulating random samples of size $100$ from several different populations where the success percentage was known and recording the percentage of successes in the sample.

|  |  |
| --- | --- |
| 1. Population with $40\%$ successes

 | 1. Population with $50\%$ successes

 |

|  |  |
| --- | --- |
| 1. Population with $60\%$ successes

 | 1. Population with $70\%$ successes

 |

* 1. Based on these graphs, which of the following are plausible values for the percentage of successes in the population from which the sample was selected: $40\%, 50\%, 60\%, $or$ 70\%$? Explain your thinking.
	2. Would you need more information to determine plausible values for the actual proportion of the population of high school students who intend to go to some postsecondary school? Why or why not?
1. Suppose the mystery bag had resulted in the following number of red chips. Using the simulated sampling distributions found earlier in this lesson, find a margin of error in each case.
	1. The number of red chips in a random sample of size $30$ was $10$.
	2. The number of red chips in a random sample of size $30$ was $21$.
	3. The number of red chips in a random sample of size $50$ was $22$.
2. The following intervals were plausible population proportions for a given sample. Find the margin of error in each case.
	1. from $0.35$ to $0.65$
	2. from $0.72$ to $0.78$
	3. from $0.84$ to $0.95$
	4. from $0.47$ to $0.57$
3. Decide if each of the following statements is true or false. Explain your reasoning in each case.
	1. The smaller the sample size, the smaller the margin of error.
	2. If the margin of error is $0.05$ and the observed proportion of red chips is $0.35$, then the true population proportion is likely to be between $0.40$ and $0.50$.
4. Extension: The margin of error for a sample of size $30$ is $0.20$; for a sample of $50$, is $0.10$. If you increase the sample size to $70$, do you think the margin of error for the percent of successes will be $0.05$? Why or why not?