Lesson 3: Calculating Conditional Probabilities and Evaluating Independence Using Two-Way Tables

Student Outcomes

- Students construct a hypothetical 1000 two-way table from given probability information and use the table to calculate the probabilities of events.
- Students calculate conditional probabilities given a two-way data table or using a hypothetical 1000 two-way table.
- Students interpret probabilities, including conditional probabilities, in context.

Lesson Notes

This lesson is a continuation of the work started in Lesson 2. In this lesson, students learn a more formal definition of conditional probability and are asked to interpret conditional probabilities. Data are presented in two-way frequency tables, and conditional probabilities are calculated using column or row summaries. The work in this lesson leads up to the definition of independent events (Lesson 4).

Classwork

Example 1 (2–3 minutes)

The initial activity asks students to investigate a real-world context using probability. In this hypothetical example, students determine if female students are more likely to be involved in an athletic program as compared to males. While questions are provided, consider using this opening problem without scaffolding to allow students to apply their thinking from the first two lessons.

Example 1

Students at Rufus King High School were discussing some of the challenges of finding space for athletic teams to practice after school. Part of the problem, according to Kristin, is that the females are more likely to be involved in after-school athletic programs than males. However, the athletic director assigns the available facilities as if males are more likely to be involved. Before suggesting changes to the assignments, the students decided to investigate.

Suppose the following information is known about Rufus King High School: 40% of students are involved in one or more of the after-school athletic programs offered at the school. It is also known that 58% of the school's students are female. The students decide to construct a hypothetical 1000 two-way table, like Table 1, to organize the data.

Scaffolding:

Discuss types of teams offered by your school and where they practice. Take a poll in the class to find out the number of males and females who participate in sports, what type, and when and where they practice. Calculate the class statistics before doing this example.

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Table 1

Participation in after-school athletic program (Yes or No) by gender

	Yes - Participate in After-School Athletic Program	No - Do Not Participate in After-School Athletic Program	Total
Females	Cell 1	Cell 2	Cell 3
Males	Cell 4	Cell 5	Cell 6
Total	Cell 7	Cell 8	Cell 9

Exercises 1–6 (10-15 minutes): Organizing the Data

MP.2 Let students work with a partner and then confirm answers as a class. Exercise 3 requires students to express their thinking, reasoning from the table abstractly and quantitatively. As students calculate the probabilities, they need to interpret them in context.

Exercises 1–6: Organizing the Data

1. **What cell in Table 1 represents a hypothetical group of $1,000$ students at Rufus King High School?**

   Cell 9

2. **What cells in Table 1 can be filled based on the information given about the student population? Place these values in the appropriate cells of the table based on this information.**

   Cells 3 and 7 can be completed from the given information. See completed table below.

3. **Based only on the cells you completed in Exercise 2, which of the following probabilities can be calculated, and which cannot be calculated? Calculate the probability if it can be calculated. If it cannot be calculated, indicate why.**

   a. **The probability that a randomly selected student is female.**

      Yes, this can be calculated. The probability is 0.58.

   b. **The probability that a randomly selected student participates in after school athletics programs.**

      Yes, this can be calculated. The probability is 0.40.

   c. **The probability that a randomly selected student who does not participate in the after school athletics program is male.**

      No, this probability cannot be calculated. We need to know the value of cell 5 to calculate this probability.

   d. **The probability that a randomly selected male student participates in the after school athletics program.**

      No, this probability cannot be calculated. We need to know the value of cell 4 to calculate this probability.

4. **The athletic director indicated that 23.2% of the students at Rufus King are females and participate in after school athletics programs. Based on this information, complete Table 1.**

   A completed table is given below:

	Yes - Participate in After-School Athletic Program	*No - Do Not Participate in After-School Athletic Program*	*Total*
Females	232	348	580
Males	168	252	420
Total	400	600	$1,000$

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5. **Consider the cells 1, 2, 4, and 5 of Table 1. Identify which of these cells represent students who are female or who participate in after-school athletic programs.**

   Cells 1, 2, and 4.

6. **What cells of the two-way table represent students who are males who do not participate in after-school athletic programs?**

   Cell 5.

Example 2 (2–3 minutes)

The following definitions were first introduced in Grade 7. It is important, however, that students revisit the definitions of complement, union, and intersection. The definitions in this lesson are connected to the context of the data and do not focus on a symbolic representation of these terms. The use of Venn diagrams and sets to represent these events is developed in Lessons 7 and 8.

Example 2

The completed hypothetical 1000 table organizes information in a way that makes it possible to answer various questions. For example, you can investigate whether females at the school are more likely to be involved in the after-school athletic programs.

Consider the following events:

- **Let "A" represent the event "a randomly selected student is female."**
- **Let "not A" represent the "complement of A." The complement of A represents the event "a randomly selected student is not female," which is equivalent to the event "a randomly selected student is male."**
- **Let "B" represent the event "a randomly selected student participates in the after-school athletic program."**
- **Let "not B" represent the "complement of B." The complement of B represents the event "a randomly selected student does not participate in the after-school athletic program."**
- **Let "A or B" (described as A union B) represent the event "a randomly selected student is female or participates in the after-school athletic program."**
- **Let "A and B" (described as A intersect B) represent the event "a randomly selected student is female and participates in the after-school athletic program."**

Exercises 7–9 (8–10 minutes)

Let students continue to work with their partner and confirm answers as a class.

Exercises 7–9

7. **Based on the descriptions above, describe the following events in words:**

   a. **Not A or Not B.**

      Males or students not participating in the after-school athletic program.

   b. **A and Not B.**

      Females not participating in the after-school athletic program.

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8. **Based on the above descriptions and Table 1, determine the probability of each of the following events:**

   a. A

   $\frac{580}{1000}$ *or* 0.58

   b. B

   $\frac{400}{1000}$ *or* 0.40

   c. Not A

   $\frac{420}{1000}$ *or* 0.42

   d. Not B

   $\frac{600}{1000}$ *or* 0.60

   e. A or B

   $\frac{232+348+168}{1000} = \frac{748}{1000}$ *or* 0.748

   f. A and B

   $\frac{232}{1000}$ *or* 0.232

9. **Determine the following values:**

   a. **The probability of A plus the probability of Not A.**

   The sum is 1.000 *or* $0.580 + 0.420 = 1.000$.

   b. **The probability of B plus the probability of Not B.**

   The sum is 1.000 *or* $0.4000 + 0.6000 = 1.000$.

   c. **What do you notice about the results of parts (a) and (b)? Explain.**

   Both probabilities total 1*. This makes sense since both parts are asking for the probability of* A *and not* A *or* B *and not* B*.*

Example 3 (2 minutes): Conditional Probability

Read through the example as a class. Help students identify how the conditional probabilities are not based on the whole population but rather on a specific subgroup within the whole population that is represented by a row total or a column total. The visual of pulling apart the two-way table by rows or columns is intended to help you develop this idea with students.

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Example 3: Conditional Probability

Another type of probability is called a conditional probability. Pulling apart the two-way table helps to define a conditional probability.

	Yes - Participate in After-School Athletic Program	No - Do Not Participate in After-School Athletic Program	Total
Females	Cell 1	Cell 2	Cell 3

Suppose that a randomly selected student is female. What is the probability that the selected student participates in the after-school athletic program? This probability is an example of what is called a conditional probability. This probability is calculated as the number of students who are female students and participate in the after-school athletic program (or the students in cell 1) divided by the total number of female students (or the students in cell 3).

Exercises 10–15 (8–10 minutes)

The following exercises are designed to have students calculate and interpret conditional probabilities based on data in two-way tables. The conditional probabilities are based on focusing on either a row or a column in the table. Let students work with a partner and confirm answers as a class. Students are calculating probabilities and connecting the probability to the context. In these questions, students must make the connection to a subgroup within the population identified by the desired conditional probability. Both the calculation of the conditional probability and its meaning in context are developed with these questions.

MP.2

Exercises 10–15

10. **The following are also examples of conditional probabilities. Answer each question.**

    a. **What is the probability that if a randomly selected student is female, she participates in the after-school athletic program?**

    $\frac{\textit{Cell } 1}{\textit{Cell } 3} = \frac{232}{580}$ *or* 0.40

    b. **What is the probability that if a randomly selected student is female, she does not participate in after-school athletics?**

    $\frac{\textit{Cell } 2}{\textit{Cell } 3} = \frac{348}{580}$ *or* 0.60

11. **Describe two conditional probabilities that can be determined from the following row in Table 1.**

	Yes - Participate in After-School Athletic Program	No - Do Not Participate in After-School Athletic Program	Total
Males	Cell 4	Cell 5	Cell 6

The probability that if a randomly selected student is male, he participates in the after-school athletic program.

The probability that if a randomly selected student is male, he does not participate in the after-school athletic program.

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12. Describe two conditional probabilities that can be determined from the following column in Table 1.

	Yes - Participate in After-School Athletic Program
Females	Cell 1
Males	Cell 4
Total	Cell 7

The probability that if a randomly selected student participates in the after-school athletic program, the student is a female.

The probability that if a randomly selected student participates in the after-school athletic program, the student is a male.

13. Determine the following conditional probabilities.

a. A randomly selected student is female. What is the probability she participates in the after-school athletic program? Explain how you determined your answer.

$$\frac{232}{580} = 0.40$$

Since it is known that the selected student is female, I looked at the row for females and used that information.

b. A randomly selected student is male. What is the probability he participates in the after-school athletic program?

$$\frac{168}{420} = 0.40$$

c. A student is selected at random. What is the probability this student participates in the after-school athletic program?

$$\frac{400}{1000} = 0.40$$

14. Based on the answers to Exercise 13, do you think that female students are more likely to be involved in after-school athletics? Explain your answer.

No, the conditional probabilities indicate males and females are equally likely to be involved in the after-school athletic program.

15. What might explain the concern females expressed in the beginning of this lesson about the problem of assigning space?

It is interesting that at this school, the probability that a randomly selected student is female is 0.58. There are more females at this school. As a result, if facilities are assigned equally (given that both females and males were found to be equally likely to be involved), the number of females involved in the after-school program is greater than the number of males and might explain the concern regarding facilities.

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Closing (5 minutes)

- What did the probabilities tell us about the students at Rufus King High School?
  - *Answers will vary. Anticipate that students might mention that there are more females than males or that males and females are equally likely to be involved in after-school athletics (based on the conditional probabilities). Use this question to help students understand the difference between probabilities based on the whole population and probabilities based on a row or column of the table (conditional probabilities).*

Ask students to summarize the main ideas of the lesson in writing or with a neighbor. Use this opportunity to informally assess student comprehension of the lesson. The Lesson Summary below offers some important ideas that should be included.

> **Lesson Summary**
>
> **Data organized in a two-way frequency table can be used to calculate probabilities. The two-way frequency tables can also be used to calculate conditional probabilities.**
>
> **In certain problems, probabilities that are known can be used to create a hypothetical 1000 two-way table. This hypothetical population of $1,000$ can be used to calculate conditional probabilities.**
>
> **Probabilities are always interpreted by the context of the data.**

Exit Ticket (5 minutes)

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Name ____ Date ____________________

Lesson 3: Calculating Conditional Probabilities and Evaluating Independence Using Two-Way Tables

Exit Ticket

A state nonprofit organization wanted to encourage its members to consider the State of New York as a vacation destination. They are investigating whether their online ad campaign influenced its members to plan a vacation in New York within the next year. The organization surveyed its members and found that 75% of them have seen the online ad. 40% of its members indicated they are planning to vacation in New York within the next year, and 15% of its members did not see the ad and do not plan to vacation in New York within the next year.

1. Complete the following hypothetical 1000 two-way frequency table:

	Plan to vacation in New York within the next year	Do not plan to vacation in New York within the next year	Total
Watched the online ad			
Did not watch the online ad			
Total			

2. Based on the two-way table, describe two conditional probabilities you could calculate to help decide if members who saw the online ad are more likely to plan a vacation in New York within the next year than those who did not see the ad.

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3. Calculate the probabilities you described in Problem 2.

4. Based on the probabilities calculated in Problem 3, do you think the ad campaign is effective in encouraging people to vacation in New York? Explain your answer.

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Exit Ticket Sample Solutions

A state nonprofit organization wanted to encourage its members to consider the State of New York as a vacation destination. They are investigating whether their online ad campaign influenced its members to plan a vacation in New York within the next year. The organization surveyed its members and found 75% of them have seen the online ad. 40% of its members indicated they are planning to vacation in New York within the next year, and 15% of its members did not see the ad and do not plan to vacation in New York within the next year.

1. Complete the following hypothetical 1000 two-way frequency table:

	Plan to vacation in New York within the next year	Do not plan to vacation in New York within the next year	Total
Watched the online ad	300	450	750
Did not watch the online ad	100	150	250
Total	400	600	$1,000$

2. Based on the two-way table, describe two conditional probabilities you could calculate to help decide if members who saw the online ad are more likely to plan a vacation in New York within the next year than those who did not see the ad.

   The probability that a randomly selected member who watched the ad is planning to vacation in New York.

   The probability that a randomly selected member who did not watch the ad is planning to vacation in New York.

3. Calculate the probabilities you described in Problem 2.

   The probability that a member who watched the ad is planning a vacation in New York is $\frac{300}{750}$ *or approximately* 0.400. *The probability that a member who did not watch the ad is planning vacation in New York is* $\frac{100}{250}$ *or approximately* 0.400.

4. Based on the probabilities calculated in Problem 3, do you think the ad campaign is effective in encouraging people to vacation in New York? Explain your answer.

   The conditional probabilities are the same. It does not appear that the ad campaign is encouraging people to vacation in New York.

Problem Set Sample Solutions

Oostburg College has a rather large marching band. Engineering majors were heard bragging that students majoring in engineering are more likely to be involved in the marching band than students from other majors.

1. If the above claim is accurate, does that mean that most of the band is engineering students? Explain your answer.

   No, it means that if a randomly selected student is an engineering major, the probability this person is in the marching band is greater than if this person was not an engineering major.

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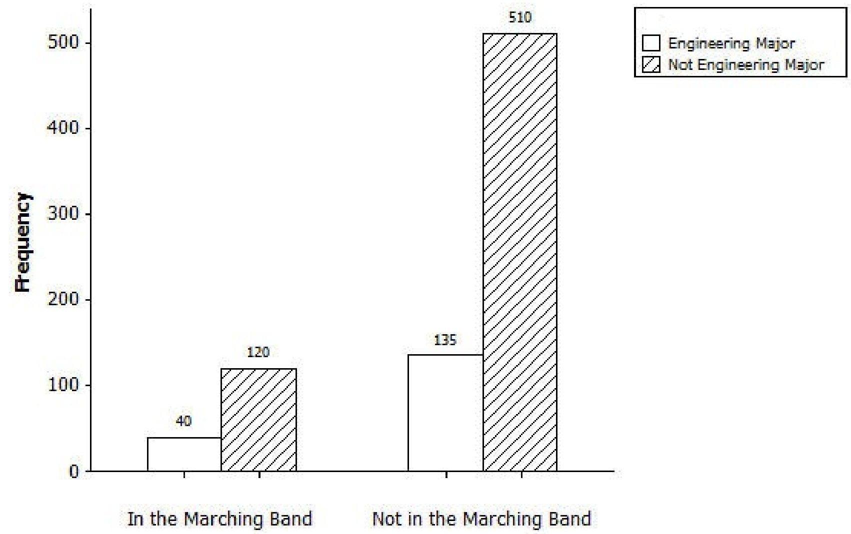
2. **The following graph was prepared to investigate the above claim.**



Based on the graph, complete the following two-way frequency table:

	In the Marching Band	Not in the Marching Band	Total
Engineering major	40	135	175
Not an engineering major	120	510	630
Total	160	645	805

3. **Let M represent the event that a randomly selected student is in the marching band. Let E represent the event that a randomly selected student is an engineering major.**

   a. **Describe the event represented by the complement of M.**

   A randomly selected student is not in the marching band.

   b. **Describe the event represented by the complement of E.**

   A randomly selected student is not majoring in engineering.

   c. **Describe the event A and B (A intersect B).**

   A randomly selected student is majoring in engineering and is in the marching band.

   d. **Describe the event A or B (A union B).**

   A randomly selected student is majoring in engineering or is in the marching band.

Note: The union can be a challenge for students to describe or to identify. Start with one of the joint cells of the two-way table. Ask if the joint cell includes students majoring in engineering or students in the marching band. If either description applies, then that cell is part of the union. Work through each of the joint cells in this way to identify all of the cells that compose the union.

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4. Based on the completed two-way frequency table, determine the following and explain how you got your answer.

   a. The probability that a randomly selected student is in the marching band.

   $\frac{160}{805} \approx 0.199$

   I compared the number of students in marching band to the total number of number of students.

   b. The probability that a randomly selected student is an engineering major.

   $\frac{175}{805} \approx 0.217$

   I compared the number of engineering majors to the total number of students.

   c. The probability that a randomly selected student is in the marching band and an engineering major.

   $\frac{40}{805} \approx 0.05$

   I found the number of students who are in the band and are engineering majors and compared it to the total number of students.

   d. The probability that a randomly selected student is in the marching band and not an engineering major.

   $\frac{120}{805} \approx 0.149$

   I found the number of students who are in the band and are NOT engineering majors and compared it to the total number of students.

5. Indicate if the following conditional probabilities would be calculated using the rows or the columns of the two-way frequency table.

   a. A randomly selected student is majoring in engineering. What is the probability this student is in the marching band?

   This probability is based on the row "Engineering major."

   b. A randomly selected student is not in the marching band. What is the probability that this student is majoring in engineering?

   This probability is based on the column "Not in the Marching Band."

6. Based on the two-way frequency table, determine the following conditional probabilities.

   a. A randomly selected student is majoring in engineering. What is the probability that this student is in the marching band?

   $\frac{40}{175} \approx 0.229$

   b. A randomly selected student is not majoring in engineering. What is the probability that this student is in the marching band?

   $\frac{120}{630} \approx 0.190$

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7. **The claim that started this investigation was that students majoring in engineering are more likely to be in the marching band than students from other majors. Describe the conditional probabilities that would be used to determine if this claim is accurate.**

   Given a randomly selected student is an engineering major, what is the probability the student is in the marching band. Also, given a randomly selected student is not an engineering major, what is the probability the student is in the marching band.

8. **Based on the two-way frequency table, calculate the conditional probabilities identified in Problem 7.**

   The probabilities were calculated in Problem 6. Approximately 0.229 (or 22.9%) of the engineering students are in the marching band. Approximately 0.190 (or 19.0%) of the students not majoring in engineering are in the marching band.

9. **Do you think the claim that students majoring in engineering are more likely to be in the marching band than students for other majors is accurate? Explain your answer.**

   The claim is accurate based on the conditional probabilities.

10. **There are 40 students at Oostburg College majoring in computer science. Computer science is not considered an engineering major. Calculate an estimate of the number of computer science majors you think are in the marching band. Explain how you calculated your estimate.**

    The probability that a randomly selected student who is not majoring engineering is in the marching band is 0.190. As result, you would estimate that 19% of the 40 computer science majors are in the marching band. This would be $40(0.190)$ or approximately 7 or 8 students.

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