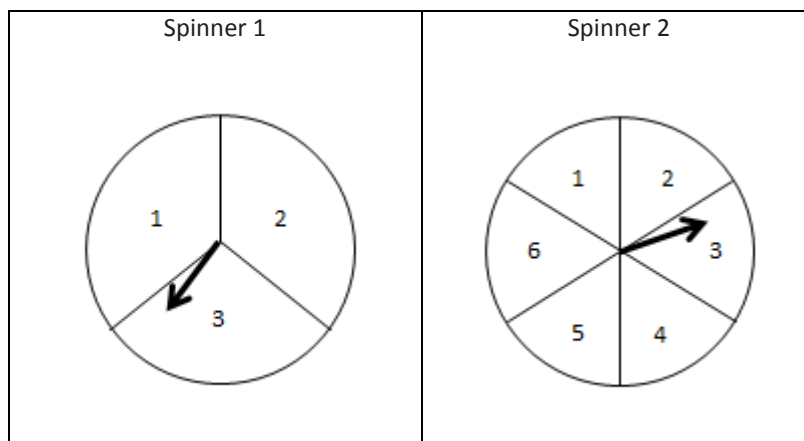


Lesson 1: Chance Experiments, Sample Spaces, and Events

Classwork

Alan is designing a probability game. He plans to present the game to people who will consider financing his idea. Here is a description of the game:

- The game includes the following materials:
 - A fair coin with a “heads” and a “tails”.
 - Spinner 1 with three equal area sectors identified as 1, 2, and 3.
 - Spinner 2 with six equal area sectors identified as 1, 2, 3, 4, 5, and 6.
 - A card bag contains six cards. Four cards are blue with the letter “A” written on one card, “B” on another card, “C” on a third card, and “D” on the fourth card. Two cards are red with the letter “E” written on one card and the letter “F” written on the other. (Although actually using colored paper is preferable, slips of paper with the words “blue” or “red” written will also work.)
 - A set of scenario cards, each describing a chance experiment and a set of five possible events based on the chance experiment.



Card bag:

Blue A	Blue B	Blue C	Blue D	Red E	Red F
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- The game is played by two players (or two small groups of players) identified as Player 1 and Player 2.
- Rules of the game:
 - The scenario cards are shuffled and one is selected.
 - Each player reads the description of the chance experiment and the description of the five possible outcomes.
 - Players independently assign the numbers 1–5 (no repeats) to the five events described on the Scenario Card based on how likely they think the event is to occur, with 5 being most likely and 1 being least likely.
 - Once players have made their assignments, the chance experiment described on the scenario card is performed. Points are then awarded based on the outcome of the chance experiment. If the event described on the scenario card has occurred, the player earns the number of points corresponding to the number that player assigned to that event (1–5 points). If an event occurs that is not described on the scenario card, then no points are awarded for that event.
 - If an outcome is described by two or more events on the scenario card, the player selects the higher point value.
 - The chance experiment is repeated four more times with points being awarded each time the chance experiment is performed.
 - The player with the largest number of points at the end of the game is the winner.

Alan developed two Scenario Cards for his demonstration to the finance people. A table in which the players can make their assignments and keep track of their scores accompanies each scenario card. Consider the first scenario card Alan developed:

Scenario Card 1

Game Tools: **Spinner 1** (three equal sectors with the number 1 in one sector, the number 2 in the second sector, and the number 3 in the third sector)
Card Bag (Blue-A, Blue-B, Blue-C, Blue-D, Red-E, Red-F)

Directions (chance experiment): Spin Spinner 1 and randomly select a card from the card bag (four blue cards and two red cards). Record the number from your spin and the color of the card selected.

Five events of interest:

Outcome is an odd number on Spinner 1 and a red card from the card bag.	Outcome is an odd number on Spinner 1.	Outcome is an odd number on Spinner 1 and a blue card from the card bag.	Outcome is an even number from Spinner 1 or a red card from the card bag.	Outcome is not a blue card from the card bag.

Player:

Scoring Card for Scenario 1:

Turn	Outcome from Spinner 1	Outcome from the card bag	Points
1			
2			
3			
4			
5			

Here is an example of Alan demonstrating the first scenario card: The chance experiment for Scenario Card 1 is: “Spin Spinner 1 and record the number. Randomly select a card from the card bag (four blue cards and two red cards). Record the color of the card selected.”

Alan assigned the numbers 1–5 to the following descriptions as shown below. Once a number is assigned, it cannot be used again.

Five events of interest:

Outcome is an odd number on Spinner 1 and a red card from the card bag.	Outcome is an odd number on Spinner 1.	Outcome is an odd number on Spinner 1 and a blue card from the card bag.	Outcome is an even number from Spinner 1 or a red card from the card bag.	Outcome is not a blue card from the card bag.
3	1	4	2	5

Alan is now ready to take his five turns. The results were recorded from the spinner and the card bag. Based on the results, Alan earned the points indicated for each turn.

Player: Player 1

Scoring Card for Scenario 1:

Turn	Outcome from Spinner 1	Outcome from the card bag	Points based on Alan’s assignment of the numbers to the five events
1	2	Blue	2
2	1	Red	5
3	1	Red	5
4	3	Blue	4
5	2	Blue	2

Alan earned a total of 18 points. The game now turns to Player 2. Player 2 assigns the numbers 1–5 to the same description of outcomes. Player 2 does not have to agree with the numbers Alan assigned. After five turns, the player with the most number of points is the winner.

Exploratory Challenge/Exercises 1–13

1. Would you change any of the assignments of 1–5 that Alan made? Explain your answer. Assign the numbers 1–5 to the event descriptions based on what you think is the best strategy to win the game.

Outcome is an odd number on Spinner 1 and a Red card from the card bag.	Outcome is an odd number on Spinner 1 .	Outcome is an odd number on Spinner 1 and a Blue card from the card bag.	Outcome is an even number from Spinner 1 or a Red card from the card bag.	Outcome is not a Blue card from the card bag.

2. Carry out a turn by observing an outcome from spinning Spinner 1 and picking a card. How many points did you earn from this first turn?

3. Complete four more turns (for a total of five) and determine your final score.

Player: Your Turn

Scoring Card for Scenario 1:

Trial	Outcome from Spinner 1	Outcome from the card bag	Points based on your assignment of numbers to the events
1			
2			
3			
4			
5			

4. If you changed the numbers assigned to the descriptions, was your score better than Alan's score? Did you expect your score to be better? Explain. If you did not change the numbers from those that Alan assigned, explain why you did not change them.

5. Spinning Spinner 1 and drawing a card from the card bag is a **chance experiment**. One possible outcome of this experiment is (1, Blue-A). Recall that the **sample space** for a chance experiment is the set of all possible outcomes. What is the sample space for the chance experiment of Scenario Card 1?
6. Are the outcomes in the sample space equally likely? Explain your answer.
7. Recall that an **event** is a collection of outcomes from the sample space. One event of interest for someone with Scenario Card 1 is “Odd number on Spinner 1 and a Red card.” What are the outcomes that make up this event? List the outcomes of this event in the first row of the Table 1 (see Exercise 9).
8. What is the probability of getting an odd number on Spinner 1 and picking a Red card from the card bag? Also enter this probability in Table 1 (see Exercise 9).

9. Complete the Table 1 by listing the outcomes for the other events and their probabilities based on the chance experiment for this scenario card.

Table 1

Event	Outcomes	Probability
Odd number on Spinner 1 and a red card from the card bag		
Odd number on Spinner 1		
Odd number on Spinner 1 and a blue card from the card bag		
Even number on Spinner 1 or a Red card from the card bag		
Not picking a blue card from the card bag		

10. Based on the above probabilities, how would you assign the numbers 1 to 5 to each of the game descriptions? Explain.
11. If you changed any of the points assigned to the game descriptions, play the game again at least three times and record your final scores for each game. Do you think you have the best possible assignment of numbers to the events for this scenario card? If you did not change the game descriptions, also play the game so that you have at least three final scores. Compare your scores with other members of your class. Do you think you have the best assignment of numbers to the events for this scenario card?

Turn	Outcome from Spinner 1	Outcome from the card bag	Points based on the assignment of points in Exercise 10
1			
2			
3			
4			
5			

12. Why might you not be able to answer the question of whether or not you have the best assignment of numbers to the game descriptions with at least three final scores?
13. Write your answers to the following questions independently, and then share your responses with a neighbor.
- How did you make decisions about what to bet on?
 - How do the ideas of probability help you make decisions?

Lesson Summary

- The sample space of a chance experiment is the collection of all possible outcomes for the experiment.
- An event is a collection of outcomes of a chance experiment.
- For a chance experiment in which outcomes of the sample space are equally likely, the probability of an event is the number of outcomes in the event divided by the number of outcomes in the sample space.
- Some events are described in terms of “or,” “and,” or “not.”

Problem Set

Consider a second scenario card that Alan created for his game:

Scenario Card 2

Tools: Spinner 1

Spinner 2: A spinner with six equal sectors. (Place the number 1 in a sector, the number 2 in a second sector, the number 3 in a third sector, the number 4 in a fourth sector, the number 5 in a fifth sector, and the number 6 in a the last sector.)

Directions (chance experiment): Spin Spinner 1, and spin Spinner 2. Record the number from Spinner 1, and record the number from Spinner 2.

Five events of interest:

Outcome is an odd number on Spinner 2.	Outcome is an odd number on Spinner 1 and an even number on Spinner 2.	Outcome is the sum of 7 from the numbers received from Spinner 1 and Spinner 2.	Outcome is an even number on Spinner 2.	Outcome is the sum of 2 from the numbers received from Spinner 1 and Spinner 2.

Player:

Scoring Card for Scenario 2:

Turn	Outcome from Spinner 1	Outcome from Spinner 2	Points
1			
2			
3			
4			
5			

1. Prepare Spinner 1 and Spinner 2 for the chance experiment described on this second scenario card. (Recall that Spinner 2 has six equal sectors.)
2. What is the sample space for the chance experiment described on this scenario card?
3. Based on the sample space, determine the outcomes and the probabilities for each of the events on this scenario card. Complete the table below.

Event	Outcomes	Probability
Outcome is an odd number on Spinner 2.		
Outcome is an odd number on Spinner 1 and an even number on Spinner 2.		
Outcome is the sum of 7 from the numbers received from Spinner 1 and Spinner 2.		
Outcome is an even number on Spinner 2.		
Outcome is the sum of 2 from the numbers received from Spinner 1 and Spinner 2.		

4. Assign the numbers 1–5 to the events described on the scenario card.

Five events of interest: Scenario Card 2

Outcome is an odd number on Spinner 2.	Outcome is an odd number on Spinner 1 and an even number on Spinner 2.	Outcome is the sum of 7 from the numbers received from Spinner 1 and Spinner 2.	Outcome is an even number on Spinner 2.	Outcome is the sum of 2 from the numbers received from Spinner 1 and Spinner 2.

5. Determine at least three final scores based on the numbers you assigned to the events.

Player: Scott

Trial	Outcome from Spinner 1	Outcome from Spinner 2	Points (see Problem 4)
1			
2			
3			
4			
5			

Player: Scott

Trial	Outcome from Spinner 1	Outcome from Spinner 2	Points (see Problem 4)
1			
2			
3			
4			
5			

Player: Scott

Trial	Outcome from Spinner 1	Outcome from Spinner 2	Points (see Problem 4)
1			
2			
3			
4			
5			

6. Alan also included a fair coin as one of the scenario tools. Develop a scenario card (Scenario Card 3) that uses the coin and one of the spinners. Include a description of the chance experiment and descriptions of five events relevant to the chance experiment.

Scenario Card 3

Tools: Fair coin (Head or Tail)
Spinner

Directions (chance experiment):

Five events of interest:

7. Determine the sample space for your chance experiment. Then, complete the table below for the five events on your scenario card. Assign the numbers 1–5 to the descriptions you created.

Event	Outcomes	Probability

8. Determine a final score for your game based on five turns.

Turn			Points
1			
2			
3			
4			
5			