Lesson 30: Buying a Car

Student Outcomes

- Students use the sum of a finite geometric series formula to develop a formula to calculate a payment plan for a car loan and use that calculation to derive the present value of an annuity formula.

Lesson Notes

In this lesson, students will explore the idea of getting a car loan. The lesson extends their knowledge on saving money from the last lesson to the mathematics behind borrowing it. The formula for the monthly payment on a loan is derived using the formula for the sum of a geometric series. Amortization tables are used to help students develop an understanding of borrowing money.

In this lesson, we derive the future amount of an annuity formula again in the context of purchasing a car and use it to understand the present value of an annuity formula,

$$A_p = R \cdot \frac{1-(1+i)^{-n}}{i}.$$

It is helpful to think of the present value of an annuity A_p in the following way: Calculate the future amount of an annuity A_f (as in Lesson 29) to find out the total amount that would be in an account after making all of the payments. Then, use the compound interest formula $F = P(1+i)^n$ from Lesson 26 to compute how much would need to be invested today (i.e., A_p) in one single large deposit to equal the amount A_f in the future. More specifically, for an interest rate of i per time period with n payments each of amount R, then the present value can be computed (substituting A_p for P and A_f for F in the compound interest formula) to be

$$A_f = A_p(1+i)^n.$$

Using the future amount of an annuity formula and solving for A_p gives

$$A_p = R \cdot \frac{(1+i)^n - 1}{i}(1+i)^{-n},$$

which simplifies to the first formula above. The play between the sum of a geometric series (**A-SSE.B.4**) and the combination of functions to get the new function A_p (**F-BF.A.1b**) constitutes the entirety of the mathematical content of this lesson.

While the mathematics is fairly simple, the context—car loans and the amortization process—is also new to students. To help the car loans process make sense (and loans in general), we have students think about the following situation: Instead of paying the full price of a car immediately, a student asks the dealer to develop a loan payment plan in which the student pays the same amount each month. The car dealer agrees and does the following calculation to determine the amount R that the student should pay each month:

- The car dealer first imagines how much she would have if she took the amount of the loan (i.e., price of the car) and deposited it into an account for 60 months (5 years) at a certain interest rate per month.

- The car dealer then imagines taking the student's payments (R dollars) and depositing them into an account making the same interest rate per month. The final amount is calculated just like calculating the final amount of a structured savings plan from Lesson 29.
- The car dealer then reasons that, to be fair to her and her customer, the two final amounts should be the same—that is, the car dealer should have the same amount in each account at the end of 60 months either way. This sets up the equation above, which can then be solved for R.

This lesson is the first lesson where the concept of amortization appears. An example of amortization is the process of decreasing the amount owed on a loan over time, which decreases the amount of interest owed over time as well. This can be thought of as doing an annuity calculation like in Lesson 29 but run backward in time. Whenever possible, use online calculators such as http://www.bankrate.com/calculators/mortgages/amortization-calculator.aspx to generate amortization tables (i.e., tables that show the amount of the principal and interest for each payment). Students have filled in a few amortization tables in Lesson 26 as an application of interest, but the concept was not presented in its entirety.

Classwork

Opening Exercise (2 minutes)

The following problem is similar to homework students did in the previous lesson; however, the savings terms are very similar to those found in car loans.

Opening Exercise

Write a sum to represent the future amount of a structured savings plan (i.e., annuity) if you deposit $\$250$ into an account each month for 5 years that pays 3.6% interest per year, compounded monthly. Find the future amount of your plan at the end of 3 years.

$250(1.003)^{59} + 250(1.003)^{58} + \cdots + 250(1.003) + 250$. *The amount in dollars in the account after 3 years will be*

$$250 \cdot \frac{(1.003)^{60} - 1}{0.003} \approx 16,407.90.$$

Example (15 minutes)

Many people take out a loan to purchase a car and then repay the loan on a monthly basis. Announce that we will figure out how banks determine the monthly loan payment for a loan in today's class.

- If you decide to get a car loan, there are many things that you will have to consider. What do you know that goes into getting a loan for a vehicle?
  - *Look for the following: down payment, a monthly payment, interest rates on the loan, number of years of the loan. Explain any of these terms that students may not know.*

For car loans, a down payment is not always required, but a typical down payment is 15% of the total cost of the vehicle. We will assume throughout this example that no down payment is required.

This example is a series of problems to work through with your students that guides students through the process for finding the recurring monthly payment for a car loan described in the teacher notes. After the example, students will be given more information on buying a car and will calculate the monthly payment for a car that they researched on the Internet as part of their homework in Lesson 29.

Example

Jack wanted to buy a $\$9,000$ 2-door sports coupe but could not pay the full price of the car all at once. He asked the car dealer if she could give him a loan where he paid a monthly payment. She told him she could give him a loan for the price of the car at an annual interest rate of 3.6% compounded monthly for 60 months (5 years).

The problems below exhibit how Jack's car dealer used the information above to figure out how much his monthly payment of R dollars per month should be.

a. **First, the car dealer imagined how much she would have in an account if she deposited $\$9,000$ into the account and left it there for 60 months at an annual interest rate of 3.6% compounded monthly. Use the compound interest formula $F = P(1+i)^n$ to calculate how much she would have in that account after 5 years. This is the amount she would have in the account after 5 years if Jack gave her $\$9,000$ for the car, and she immediately deposited it.**

$F = 9000(1+0.003)^{60} = 9000(1.003)^{60} \approx 10,772.05$. *At the end of 60 months, she would have $\$10,772.05$ in the account.*

b. **Next, she figured out how much would be in an account after 5 years if she took each of Jack's payments of R dollars and deposited it into a bank that earned 3.6% per year (compounded monthly). Write a sum to represent the future amount of money that would be in the annuity after 5 years in terms of R, and use the sum of a geometric series formula to rewrite that sum as an algebraic expression.**

This is like the structured savings plan in Lesson 29. The future amount of money in the account after 5 years can be represented as

$$R(1.003)^{59} + R(1.003)^{58} + \cdots + R(1.003) + R.$$

Applying the sum of a geometric series formula

$$S_n = a \cdot \frac{1-r^n}{1-r}$$

to the geometric series above using $a = R$, $r = 1.003$, and $n = 60$, one gets

$$S_n = R \cdot \frac{1-(1.003)^{60}}{1-1.003} = R \cdot \frac{(1.003)^{60}-1}{0.003}.$$

At this point, we have re-derived the future amount of an annuity formula. Point this out to your students! Help them to see the connection between what they are doing in this context with what they did in Lesson 29. The future value formula is

$$A_f = R \cdot \frac{(1+i)^n - 1}{i}.$$

c. **The car dealer then reasoned that, to be fair to her and Jack, the two final amounts in both accounts should be the same—that is, she should have the same amount in each account at the end of 60 months either way. Write an equation in the variable R that represents this equality.**

$$9000(1.003)^{60} = R \cdot \frac{(1.003)^{60}-1}{0.003}$$

d. **She then solved her equation to get the amount R that Jack would have to pay monthly. Solve the equation in part (c) to find out how much Jack needed to pay each month.**

Solving for R in the equation above, we get

$$R = 9000 \cdot (1.003)^{60} \cdot \frac{0.003}{(1.003)^{60} - 1} \approx 164.13.$$

Thus, Jack will need to make regular payments of $\$164.13$ a month for 60 months.

Ask students questions to see if they understand what the $164.13 means. For example, if Jack decided not to buy the car and instead deposited $164.13 a month into an account earning 3.6% interest compounded monthly, how much will he have at the end of 60 months? Students should be able to answer $10,772.05, the final amount of the annuity that the car dealer calculated in part (a) (or (b)). Your goal is to help them see that both ways of calculating the future amount should be equal.

Discussion (10 minutes)

In this discussion, students are lead to the present value of an annuity formula using the calculations they just did in the example (**F-BF.A.1b**).

MP.8

- Let's do the calculations in part (a) of the example again but this time using A_p for the loan amount (the present value of an annuity), i for the interest rate per time period, n to be the number of time periods. As in part (a), what is the future value of A_p if it is deposited in an account with an interest rate of i per time period for n compounding periods?
  - $F = A_p(1+i)^n$
- As in part (b) of the example above, what is the future value of an annuity A_f in terms of the recurring payment R, interest rate i, and number of periods n?
  - $A_f = R \cdot \frac{1-(1+i)^n}{i}$
- If we assume (as in the example above) that both methods produce the same future value, we can equate $F = A_f$ and write the following equation:

$$A_p(1+i)^n = R \cdot \frac{(1+i)^n - 1}{i}.$$

- What equation is this in example above?
  - *The equation derived in part (c).*
- We can now solve this equation for R as we did in the example, but it is more common in finance to solve for A_p by multiplying both sides by $(1+i)^{-n}$:

$$A_p = R \cdot \frac{(1+i)^n - 1}{i} \cdot (1+i)^{-n},$$

  and then distributing it through the binomial to get the *present value of an annuity* formula:

$$A_p = R \cdot \frac{1-(1+i)^{-n}}{i}.$$

- When a bank (or a car dealer) makes a loan that is to be repaid with recurring payments R, then the payments form an annuity whose present value A_p is the amount of the loan. Thus, we can use this formula to find the payment amount R given the size of the loan A_p (as in Example 1), or we can find the size of the loan A_p if we know the size of the payments R.

Exercise (3 minutes)

Exercise

A college student wants to buy a car and can afford to pay $\$200$ per month. If she plans to take out a loan at 6% interest per year with a recurring payment of $\$200$ per month for four years, what price car can she buy?

$$A_p = 200 \cdot \frac{1-(1.005)^{-48}}{0.005} \approx 8,516.06$$

She can afford to take out a $\$8,516.06$ loan. If she has no money for a down payment, she can afford a car that is about $\$8,500$.

You might want to point out to your students that the present value formula can always be easily and quickly derived from the future amount of annuity formula $A_f = R \cdot \frac{1-(1+i)^n}{i}$ and the compound interest formula $A_f = A_p(1+i)^n$ (using the variables A_f and A_p instead of F and P).

Mathematical Modeling Exercise (8 minutes)

The customization and open-endedness of this challenge depends upon how successful students were in researching the price of a potential car in the Problem Set to Lesson 29. For students who did not find a car, you can have them use the list provided below. After the challenge, there are some suggestions for ways to introduce other modeling elements into the challenge. Use the suggestions as you see fit. The solutions throughout this section are based on the 2007 two-door small coupe.

MP.2 & MP.4

Mathematical Modeling Exercise

In the Problem Set of Lesson 29, you researched the price of a car that you might like to own. In this exercise, you will determine how much a car payment would be for that price for different loan options.

If you did not find a suitable car, select a car and selling price from the list below:

Car	Selling Price
2005 Pickup Truck	$\$9,000$
2007 Two-Door Small Coupe	$\$7,500$
2003 Two-Door Luxury Coupe	$\$10,000$
2006 Small SUV	$\$8,000$
2008 Four-Door Sedan	$\$8,500$

a. **When you buy a car, you must pay sales tax and licensing and other fees. Assume that sales tax is 6% of the selling price and estimated license/title/fees will be 2% of the selling price. If you put a $\$1,000$ down payment on your car, how much money will you need to borrow to pay for the car and taxes and other fees?**

Answers will vary. For the 2007 two-door small coupe: $7500 + 7500(0.06) + 7500(0.02) - 1000 = 7100$

You would have to borrow $\$7,100$.

Scaffolding:

For English Language Learners, provide a visual image of each vehicle type along with a specific make and model.

- Pickup Truck
- 2-Door Small Coupe
- 2-Door Luxury Coupe
- Small SUV
- 4-Door Sedan

b. Using the loan amount you computed above, calculate the monthly payment for the different loan options shown below:

Loan 1	36-month loan at 2%
Loan 2	48-month loan at 3%
Loan 3	60-month loan at 5%

Answers will vary. For the 2007 two-door small coupe:

Loan 1: $7100 = R \cdot \frac{1-\left(1+\frac{0.02}{12}\right)^{-36}}{\frac{0.02}{12}}$*; therefore,* $R \approx 203.36$*. The monthly payment would be* $\$203.36$*.*

Loan 2: $7100 = R \cdot \frac{1-\left(1+\frac{0.03}{12}\right)^{-48}}{\frac{0.03}{12}}$*; therefore,* $R \approx 157.15$*. The monthly payment would be* $\$157.15$*.*

Loan 3: $7100 = R \cdot \frac{1-\left(1+\frac{0.05}{12}\right)^{-60}}{\frac{0.05}{12}}$*; therefore,* $R \approx 133.99$*. The monthly payment would be* $\$133.99$*.*

c. Which plan, if any, will keep your monthly payment under $\$175$? Of the plans under $\$175$ per month, why might you choose a plan with fewer months even though it costs more per month?

Answers will vary. Loan 2 and Loan 3 are both under $\$175$ a month. When the monthly payments are close (like Loan 2 and Loan 3), the fewer payments you make with Loan 2 means you pay less overall for that loan.

If a student found a dealer that offered a loan for the car they were researching, encourage them to do the calculations above for terms of that loan. (Call it loan option 4.)

Further Modeling Resources

If students are interested in the actual details of purchasing and budgeting for a car, the following websites can be referenced for further exploration.

Vehicle Fees: http://www.dmv.org/ny-new-york/car-registration.php

Inspection: http://dmv.ny.gov/forms/vs77.pdf

Car Maintenance: http://www.edmunds.com/tco.html (2009 and newer models only) or http://www.edmunds.com/calculators/

Car Insurance: http://dmv.ny.gov/insurance/looking-insurance-information

Furthermore, you can ask:

- What sort of extra fees go into buying a car?
  - *Answers may vary, but students should be able to come up with sales tax, which is 4% in New York State. Additional sales taxes may also apply for your local jurisdiction. Other fees include license plate ($\$25$) and title ($\$50$). An inspection is also required within 10 days of purchase ($\$10$). Other states may have additional fees.*
- What extra costs go into maintaining a car?
  - *Insurance, repairs, maintenance like oil changes, car washing/detailing, etc. For used and new cars, use an online calculator to estimate car insurance and maintenance costs as well as likely depreciation and interest costs for a loan.*

Closing (2 minutes)

Close this lesson by asking students to summarize in writing or with a partner what they know so far about borrowing money to buy a car.

- Based on the work you did in this lesson, summarize what you know so far about borrowing money to buy a car.
  - *Making the loan term longer does make the monthly payment go down but causes the total interest paid to go up. Interest rates, down payment, and total length of the loan all affect the monthly payment. In the end, the amount of the loan you get depends on what you can afford to pay per month based on your budget.*

Lesson Summary

The total cost of car ownership includes many different costs in addition to the selling price, such as sales tax, insurance, fees, maintenance, interest on loans, gasoline, etc.

The present value of an annuity formula can be used to calculate monthly loan payments given a total amount borrowed, the length of the loan, and the interest rate. The present value A_p (i.e., loan amount) of an annuity consisting of n recurring equal payments of size R and interest rate i per time period is

$$A_p = R \cdot \frac{1-(1+i)^{-n}}{i}.$$

Amortization tables and online loan calculators can also help you plan for buying a car.

The amount of your monthly payment depends on the interest rate, the down payment, and the length of the loan.

Exit Ticket (5 minutes)

Name ______________________________ Date ______________

Lesson 30: Buying a Car

Exit Ticket

Fran wants to purchase a new boat. She starts looking for a boat around $\$6{,}000$. Fran creates a budget and thinks that she can afford $\$250$ every month for 2 years. Her bank charges her 5% interest per year, compounded monthly.

1. What is the actual monthly payment for Fran's loan?

2. If Fran can only pay $\$250$ per month, what is the most expensive boat she can buy without a down payment?

Exit Ticket Sample Solutions

Fran wants to purchase a new boat. She starts looking for a boat around $\$6,000$. Fran creates a budget and thinks that she can afford $\$250$ every month for 2 years. Her bank charges her 5% interest per year, compounded monthly.

1. What is the actual monthly payment for Fran's loan?

$$6000 = R\left(\frac{1-\left(1+\frac{0.05}{12}\right)^{-24}}{\frac{0.05}{12}}\right)$$

$$R = 6000\left(\frac{\left(\frac{0.05}{12}\right)}{1-\left(1+\frac{0.05}{12}\right)^{-24}}\right)$$

$$R \approx \$263.23$$

2. If Fran can only pay $\$250$ per month, what is the most expensive boat she can buy without a down payment?

$$P = 250\left(\frac{1-\left(1+\frac{0.05}{12}\right)^{-24}}{\frac{0.05}{12}}\right)$$

$$P \approx \$5698.47$$

Fran can afford a boat that costs about $\$5,700$ if she does not have a down payment.

Problem Set Sample Solutions

1. Benji is 24 years old and plans to drive his new car about 200 miles per week. He has qualified for first-time buyer financing, which is a 60-month loan with 0% down at an interest rate of 4%. Use the information below to estimate the monthly cost of each vehicle.

CAR A: 2010 Pickup Truck for $\$12,000$, 22 miles per gallon

CAR B: 2006 Luxury Coupe for $\$11,000$, 25 miles per gallon

Gasoline: $\$4.00$ per gallon — New vehicle fees: $\$80$ — Sales Tax: 4.25%

Maintenance Costs:

(2010 model year or newer): 10% of purchase price annually

(2009 model year or older): 20% of purchase price annually

Insurance:

Average Rate Ages 25–29	$\$100$ per month
If you are male If you are female	Add $\$10$ per month Subtract $\$10$ per month
Type of Car Pickup Truck Small Two-Door Coupe or Four-Door Sedan Luxury Two- or Four-Door Coupe	 Subtract $\$10$ per month Subtract $\$10$ per month Add $\$15$ per month
Ages 18–25	Double the monthly cost

a. How much money will Benji have to borrow to purchase each car?

$\$12,000$ for the truck and $\$11,000$ for the coupe.

b. What is the monthly payment for each car?

$$12000 = R\left(\frac{1-\left(1+\frac{0.04}{12}\right)^{-60}}{\frac{0.04}{12}}\right)$$

$$R = 12000\left(\frac{\left(\frac{0.04}{12}\right)}{1-\left(1+\frac{0.04}{12}\right)^{-60}}\right)$$

$$R \approx 221.00$$

The truck would cost $\$221.00$ every month.

$$11000 = R\left(\frac{1-\left(1+\frac{0.04}{12}\right)^{-60}}{\frac{0.04}{12}}\right)$$

$$R = 11000\left(\frac{\left(\frac{0.04}{12}\right)}{1-\left(1+\frac{0.04}{12}\right)^{-60}}\right)$$

$$R \approx 202.58$$

The coupe would cost $\$202.58$ every month.

c. What are the annual maintenance costs and insurance costs for each car?

Truck: $10\% \cdot 12000 = 1200$ for the maintenance. Insurance will vary based on the gender of student. Male students will be $\$200$ per month or $\$2400$ per year, while female students will be $\$160$ per month or $\$1,920$ per year.

Car: $20\% \cdot 11000 = 2200$ for maintenance. Male students will cost $\$250$ per month or $\$3,000$ per year, while female students will cost $\$210$ per month or $\$2,520$ per year.

d. Which car should Benji purchase? Explain your choice.

Answers will vary depending on personal preference and experience, as well as financial backgrounds. Answers should be supported using the mathematics of parts (a), (b), and (c).

2. Use the total initial cost of buying your car from the lesson to calculate the monthly payment for the following loan options.

Option	Number of Months	Down Payment	Interest Rate	Monthly Payment
Option A	48 months	$\$0$	2.5%	$\$175.31$
Option B	60 months	$\$500$	3.0%	$\$134.77$
Option C	60 months	$\$0$	4.0%	$\$147.33$
Option D	36 months	$\$1,000$	0.9%	$\$197.15$

Answers will vary. Suggested answers assume an $\$8,000$ car.

a. **For each option, what is the total amount of money you will pay for your vehicle over the life of the loan?**

Option A: $175.31 \cdot 48 = \$8414.88$

Option B: $500 + 134.77 \cdot 60 = \$8586.20$

Option C: $147.33 \cdot 60 = \$8839.80$

Option D: $1000 + 197.15 \cdot 36 = \$8097.40$

b. **Which option would you choose? Justify your reasoning.**

Answers will vary. Option B is the cheapest per month but requires a down payment. Of the plans without down payments, Option A saves the most money in the end, but Option C is cheaper per month. Option D saves the most money long term but requires the largest down payment and the largest monthly payment.

3. **Many lending institutions will allow you to pay additional money toward the principal of your loan every month. The table below shows the monthly payment for an $\$8,000$ loan using Option A above if you pay an additional $\$25$ per month.**

Month/ Year	Payment	Principal Paid	Interest Paid	Total Interest	Balance
Aug. 2014	$ 200.31	$ 183.65	$ 16.67	$ 16.67	$ 7,816.35
Sept. 2014	$ 200.31	$ 184.03	$ 16.28	$ 32.95	$ 7,632.33
Oct. 2014	$ 200.31	$ 184.41	$ 15.90	$ 48.85	$ 7,447.91
Nov. 2014	$ 200.31	$ 184.80	$ 15.52	$ 64.37	$ 7,263.12
Dec. 2014	$ 200.31	$ 185.18	$ 15.13	$ 79.50	$ 7,077.94
Jan. 2015	$ 200.31	$ 185.57	$ 14.75	$ 94.25	$ 6,892.37
Feb. 2015	$ 200.31	$ 185.95	$ 14.36	$ 108.60	$ 6,706.42
Mar. 2015	$ 200.31	$ 186.34	$ 13.97	$ 122.58	$ 6,520.08
April 2015	$ 200.31	$ 186.73	$ 13.58	$ 136.16	$ 6,333.35
May 2015	$ 200.31	$ 187.12	$ 13.19	$ 149.35	$ 6,146.23
June 2015	$ 200.31	$ 187.51	$ 12.80	$ 162.16	$ 5,958.72
July 2015	$ 200.31	$ 187.90	$ 12.41	$ 174.57	$ 5,770.83
Aug. 2015	$ 200.31	$ 188.29	$ 12.02	$ 186.60	$ 5,582.54
Sept. 2015	$ 200.31	$ 188.68	$ 11.63	$ 198.23	$ 5,393.85
Oct. 2015	$ 200.31	$ 189.08	$ 11.24	$ 209.46	$ 5,204.78
Nov. 2015	$ 200.31	$ 189.47	$ 10.84	$ 220.31	$ 5,015.31
Dec. 2015	$ 200.31	$ 189.86	$ 10.45	$ 230.75	$ 4,825.45

Note: The months from January 2016 to December 2016 are not shown.

Jan. 2017	$ 200.31	$ 195.07	$ 5.24	$ 330.29	$ 2,320.92
Feb. 2017	$ 200.31	$ 195.48	$ 4.84	$ 335.12	$ 2,125.44
Mar. 2017	$ 200.31	$ 195.88	$ 4.43	$ 339.55	$ 1,929.56
April 2017	$ 200.31	$ 196.29	$ 4.02	$ 343.57	$ 1,733.27
May 2017	$ 200.31	$ 196.70	$ 3.61	$ 347.18	$ 1,536.57
June 2017	$ 200.31	$ 197.11	$ 3.20	$ 350.38	$ 1,339.45
July 2017	$ 200.31	$ 197.52	$ 2.79	$ 353.17	$ 1,141.93
Aug. 2017	$ 200.31	$ 197.93	$ 2.38	$ 355.55	$ 944.00
Sept. 2017	$ 200.31	$ 198.35	$ 1.97	$ 357.52	$ 745.65
Oct. 2017	$ 200.31	$ 198.76	$ 1.55	$ 359.07	$ 546.90
Nov. 2017	$ 200.31	$ 199.17	$ 1.14	$ 360.21	$ 347.72
Dec. 2017	$ 200.31	$ 199.59	$ 0.72	$ 360.94	$ 148.13
Jan. 2018	$ 148.44	$ 148.13	$ 0.31	$ 361.25	$ 0.00

How much money would you save over the life of an $\$8,000$ loan using Option A if you paid an extra $\$25$ per month compared to the same loan without the extra payment toward the principal?

Using Option A without paying extra toward the principal each month is a monthly payment of $\$175.31$. The total amount you will pay is $\$8,414.88$. If you pay the extra $\$25$ per month, you make 41 payments of $\$200.31$ and a final payment of $\$148.44$ for a total amount of $\$8,361.15$. You would save $\$53.73$.

4. **Suppose you can afford only $\$200$ a month in car payments, and your best loan option is a 60-month loan at 3%. How much money could you spend on a car? That is, calculate the present value of the loan with these conditions.**

$$P = 200\left(\frac{1-\left(1+\frac{0.03}{12}\right)^{-60}}{\frac{0.03}{12}}\right)$$

$$P \approx 11130.47$$

You can afford a loan of about $\$11,000$. If there is no down payment, then the car would need to cost about $\$11,000$.

5. **Would it make sense for you to pay an additional amount per month toward your car loan? Use an online loan calculator to support your reasoning.**

While pre-paying on a loan can save you money for a relatively short-term loan like a vehicle loan, there is usually not a significant cost savings. Most students will probably elect to pocket the extra monthly costs and pay slightly more over the life of the loan. One option is paying off a loan early. That can save you more money and can be explored online as an extension question for advanced learners.

6. **What is the sum of each series?**
   a. $900 + 900(1.01)^1 + 900(1.01)^2 + \cdots 900(1.01)^{59}$

$$900\left(\frac{1-(1.01)^{60}}{1-1.01}\right) \approx 73502.703$$

b. $\sum_{n=0}^{47} 15,000\left(1+\frac{0.04}{12}\right)^n$

$$\sum_{n=0}^{47} 15,000\left(1+\frac{0.04}{12}\right)^n = 15000\left(\frac{1-\left(1+\frac{0.04}{12}\right)^{48}}{1-\left(1+\frac{0.04}{12}\right)}\right)$$
$$= 15000\left(\frac{\left(1+\frac{0.04}{12}\right)^{48}-1}{\frac{0.04}{12}}\right)$$
$$\approx \$779394.015$$

7. Gerald wants to borrow $\$12,000$ in order to buy an engagement ring. He wants to repay the loan by making monthly installments for two years. If the interest rate on this loan is $9\frac{1}{2}\%$ per year, compounded monthly, what is the amount of each payment?

$$12000 = R\left(\frac{1-\left(1+\frac{0.095}{12}\right)^{-24}}{\frac{0.095}{12}}\right)$$
$$R = 12000\left(\frac{\left(\frac{0.095}{12}\right)}{1-\left(1+\frac{0.95}{12}\right)^{-24}}\right)$$
$$R \approx 550.97$$

Gerald will need to pay $\$550.97$ *each month.*

8. Ivan plans to surprise his family with a new pool using his Christmas bonus of $\$4,200$ as a down payment. If the price of the pool is $\$9,500$ and Ivan can finance it at an interest rate of $2\frac{7}{8}\%$ per year, compounded quarterly, how long is the loan for if he pays $\$285.45$ per quarter?

$$5300 = 285.45\left(\frac{1-\left(1+\frac{0.02875}{4}\right)^{-n}}{\frac{0.02875}{4}}\right)$$
$$\frac{5300}{285.45}\cdot\frac{0.02875}{4} = 1-\left(1+\frac{0.02875}{4}\right)^{-n}$$
$$\left(1+\frac{0.02875}{4}\right)^{-n} = 1-\frac{5300}{285.45}\cdot\frac{0.02875}{4}$$
$$-n\cdot\ln\left(1+\frac{0.02875}{4}\right) = \ln\left(1-\frac{5300}{285.45}\cdot\frac{0.02875}{4}\right)$$
$$n = -\frac{\ln\left(1-\frac{5300}{285.45}\cdot\frac{0.02875}{4}\right)}{\ln\left(1+\frac{0.02875}{4}\right)}$$
$$n \approx 20$$

It will take Ivan 20 *quarters, or five years, to pay off the pool at this rate.*

9. Jenny wants to buy a car by making payments of $\$120$ per month for three years. The dealer tells her that she will need to put a down payment of $\$3,000$ on the car in order to get a loan with those terms at a 9% interest rate per year, compounded monthly. How much is the car that Jenny wants to buy?

$$P - 3000 = 120\left(\frac{1-\left(1+\frac{0.09}{12}\right)^{-36}}{\frac{0.09}{12}}\right)$$

$$P \approx 3773.62 + 3000$$

The car Jenny wants to buy is about $\$6,773.62$.

10. Kelsey wants to refinish the floors in her house and estimates that it will cost $\$39,000$ to do so. She plans to finance the entire amount at $3\frac{1}{4}\%$ interest per year, compounded monthly for 10 years. How much is her monthly payment?

$$39000 = R\left(\frac{1-\left(1+\frac{0.0325}{12}\right)^{-120}}{\frac{0.0325}{12}}\right)$$

$$R = 39000\left(\frac{\left(\frac{0.0325}{12}\right)}{1-\left(1+\frac{0.0325}{12}\right)^{-120}}\right)$$

$$R \approx 381.10$$

Kelsey will have to pay $\$381.10$ every month.

11. Lawrence coaches little league baseball and needs to purchase all new equipment for his team. He has $\$489$ in donations, and the team's sponsor will take out a loan at $4\frac{1}{2}\%$ interest per year, compounded monthly for one year, paying up to $\$95$ per month. What is the most that Lawrence can purchase using the donations and loan?

$$P - 489 = 95\left(\frac{1-\left(1+\frac{0.045}{12}\right)^{-12}}{\frac{0.045}{12}}\right)$$

$$P \approx 489 + 1112.69$$

The team will have access to $\$1,601.69$.