Topic E:

Geometric Series and Finance

A-SSE.B.4, F-IF.C.7e, F-IF.C.8b, F-IF.C.9, F-BF.A.1b, F-BF.A.2, F-LE.B.5

Focus Standards:	A-SSE.B.4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*
	F-IF.C.7e	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.[★]
		e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
	F-IF.C.8b	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
		b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{\frac{t}{10}}$, and classify them as representing exponential growth or decay.*
	F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
	F-BF.A.1b	Write a function that describes a relationship between two quantities.[★]
		b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
	F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.[★]
	F-LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context.

Instructional Days:	5
Lesson 29:	The Mathematics Behind a Structured Savings Plan (M)[1]
Lesson 30:	Buying a Car (M)
Lesson 31:	Credit Cards (M)
Lesson 32:	Buying a House (M)
Lesson 33:	The Million Dollar Problem (M)

Topic E is a culminating series of lessons driven by MP.4, Modeling with Mathematics. Students apply what they have learned about mathematical models and exponential growth to financial literacy, while developing and practicing the formula for the sum of a finite geometric series. Lesson 29 develops the future value formula for a structured savings plan and, in the process, develops the formula for the sum of a finite geometric series (**A-SSE.B.4**). The summation symbol, Σ, is introduced in this lesson.

Lesson 30 introduces loans through the context of purchasing a car. To develop the formula for the present value of an annuity, students combine two formulas for the future value of the annuity (**F-BF.A.1b**) and apply the sum of a finite geometric series formula. Throughout the remaining lessons, various forms of the present value of an annuity formula are used to calculate monthly payments and loan balances. Students compare the effects of various interest rates and repayment schedules, which requires that students translate between symbolic and numerical representations of functions (**F-IF.C.9**). Lesson 31 addresses the issue of revolving credit such as credit cards, for which the borrower can choose how much of the debt to pay each cycle. Students again sum a geometric series to develop a formula for this scenario, and it turns out to be equivalent to the formula used for car loans. Key features of tables and graphs are used to answer questions about finances (**F-IF.C.7e**).

Lessons 32 and 33 are modeling lessons in which students apply what they have done in earlier lessons to new financial situations (MP.4). Lesson 32 may be extended to an open-ended project in which students research buying a home and justify its affordability. Students graph the present value function and compare that with an amortization table, in accordance with **F-IF.C.9**. Lesson 33, the final lesson of the module, is primarily a summative lesson in which students formulate a plan to have $1,000,000 in assets within a fixed time frame, using the formulas developed in the prior lessons in the topic. In both of these lessons, students need to combine functions using standard arithmetic operations (**F-IF.A.1b**).

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[1] Lesson Structure Key: **P**-Problem Set Lesson, **M**-Modeling Cycle Lesson, **E**-Exploration Lesson, **S**-Socratic Lesson