Lesson 25: Geometric Sequences and Exponential Growth and Decay

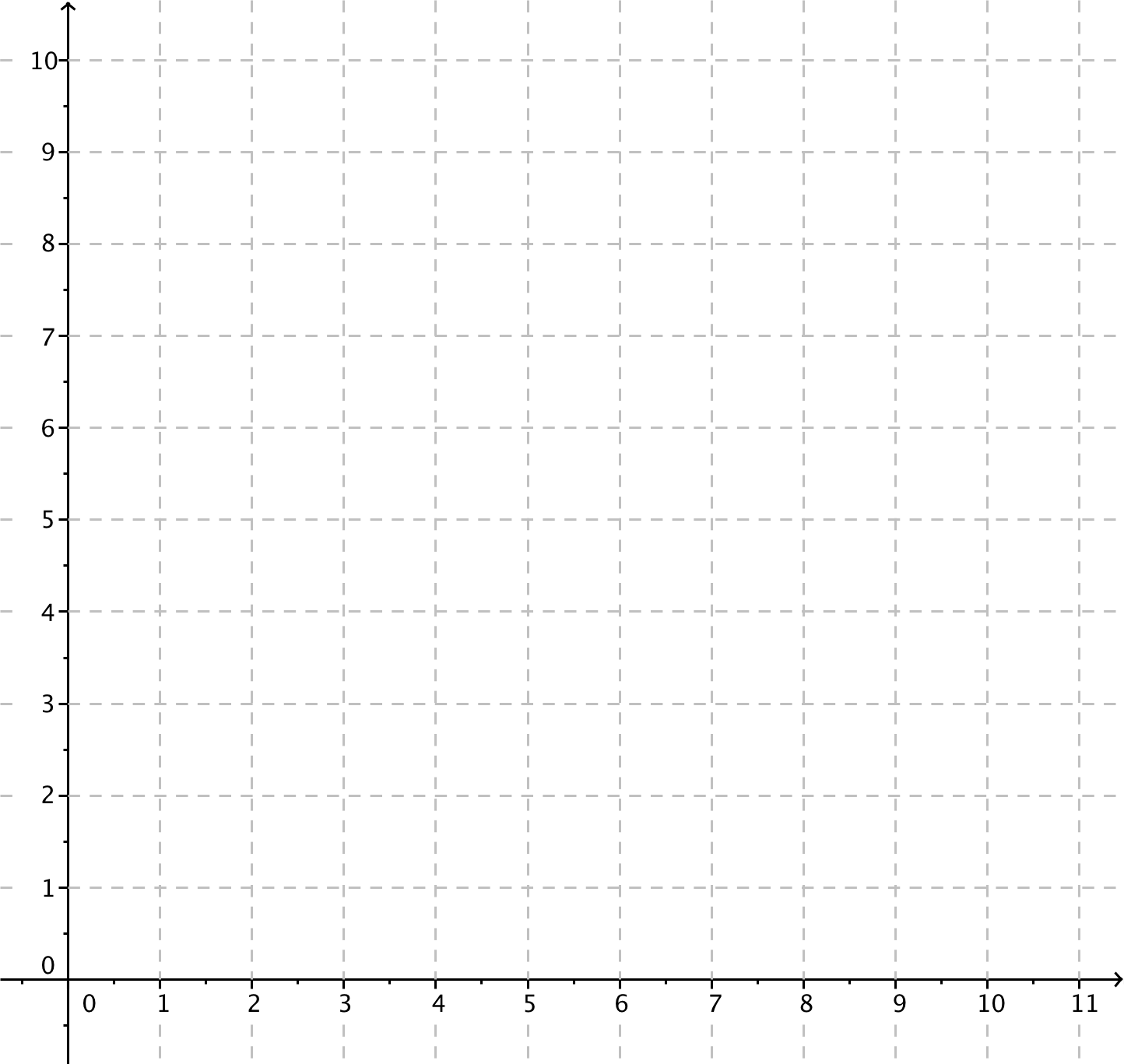
Classwork

Opening Exercise

Suppose a ball is dropped from an initial height and that each time it rebounds, its new height is of its previous height.

* 1. What are the first four rebound heights , , , and after being dropped from a height of ?
  2. Suppose the initial height is . What are the first four rebound heights? Fill in the following table:

|  |  |
| --- | --- |
| Rebound | Height (ft.) |
|  |  |
|  |  |
|  |  |
|  |  |

* 1. How is each term in the sequence related to the one that came before it?
  2. Suppose the initial height is and that each rebound, rather than being of the previous height, is times the previous height, where . What are the first four rebound heights? What is the th rebound height?
  3. What kind of sequence is the sequence of rebound heights?
  4. Suppose that we define a function with domain all real numbers so that is the first rebound height, is the second rebound height, and continuing so that is the th rebound height for positive integers . What type of function would you expect to be?
  5. On the coordinate plane below, sketch the height of the bouncing ball when and , assuming that the highest points occur at , , , , ….
  6. Does the exponential function for real numbers model the height of the bouncing ball? Explain how you know.
  7. What does the function for integers model?

Exercises

* 1. Jane works for a video game development company that pays her a starting salary of a day, and each day she works, she earns more than the day before. How much does she earn on day ?
  2. If you were to graph the growth of her salary for the first days she worked, what would the graph look like?
  3. What kind of sequence is the sequence of Jane’s earnings each day?

1. A laboratory culture begins with bacteria at the beginning of the experiment, which we will denote by time   
    hours. By time hours, there were bacteria.
   1. If the number of bacteria is increasing by a common factor each hour, how many bacteria were there at time   
       hour? At time hours?
   2. Find the explicit formula for term of the sequence in this case.
   3. How would you find term if you know term ? Write a recursive formula for in terms of .
   4. If is the initial population, the growth of the population at time hours can be modeled by the sequence , where is an exponential function with the following form:

, where .

Find the value of and write the function in this form. Approximate to four decimal places.

* 1. Use the function in part (d) to determine the value of when the population of bacteria has doubled.
  2. If is the initial population, the growth of the population at time can be expressed in the following form:

, where .

Find the value of and write the function in this form. Approximate to four decimal places.

* 1. Use the formula in part (d) to determine the value of when the population of bacteria has doubled.

1. The first term of a geometric sequence is, and the common ratio is
   1. What are the terms , and ?
   2. Find a recursive formula for this sequence.
   3. Find an explicit formula for this sequence.
   4. What is term ?
   5. What is term ?
2. Term of a geometric sequence is , and term is .
   1. What is the common ratio ?
   2. What is term ?
   3. Find a recursive formula for this sequence.
   4. Find an explicit formula for this sequence.
3. The recursive formula for a geometric sequence is with . Find an explicit formula for this sequence.
4. The explicit formula for a geometric sequence is . Find a recursive formula for this sequence.

Lesson Summary

**Arithmetic Sequence:** A sequence is called *arithmetic* if there is a real number such that each term in the sequence is the sum of the previous term and .

* *Explicit formula:* Term of an arithmetic sequence with first term and common difference is given by for .
* *Recursive formula:*  Term of an arithmetic sequence with first term and common difference is given by , for .

**Geometric Sequence:** A sequence is called *geometric* if there is a real number such that each term in the sequence is a product of the previous term and .

* *Explicit formula:*  Term of a geometric sequence with first term and common ratio is given by , for .
* *Recursive formula:*  Term of a geometric sequence with first term and common ratio is given by .

Problem Set

1. Convert the following recursive formulas for sequences to explicit formulas.
   1. with
   2. with
   3. with
   4. with
   5. with
2. Convert the following explicit formulas for sequences to recursive formulas.
   1. for
   2. for
   3. for
   4. for
   5. for
3. If a geometric sequence has and , find the exact value of the common ratio .
4. If a geometric sequence has and , approximate the value of the common ratio to four decimal places.
5. Find the difference between the terms of an arithmetic sequence and a geometric sequence, both of which begin at term and have and .
6. Given the geometric series defined by the following values of and , find the value of so that has the specified value.
   1. , ,
   2. , ,
   3. , ,
   4. , ,
7. Jenny planted a sunflower seedling that started out cm tall, and she finds that the average daily growth is .
   1. Find a recursive formula for the height of the sunflower plant on day .
   2. Find an explicit formula for the height of the sunflower plant on day .
8. Kevin modeled the height of his son (in inches) at age years for , , … , by the sequence  
    . Interpret the meaning of the constants and in his model.
9. Astrid sells art prints through an online retailer. She charges a flat rate per order for an order processing fee, sales tax, and the same price for each print. The formula for the cost of buying prints is given by  
   .
   1. Interpret the number in the context of this problem.
   2. Interpret the number in the context of this problem.
   3. Find a recursive formula for the cost of buying prints.
10. A bouncy ball rebounds to of the height of the preceding bounce. Craig drops a bouncy ball from a height of
    1. Write out the sequence of the heights , , , and of the first four bounces, counting the initial height as .
    2. Write a recursive formula for the rebound height of a bouncy ball dropped from an initial height of
    3. Write an explicit formula for the rebound height of a bouncy ball dropped from an initial height of
    4. How many bounces will it take until the rebound height is under ?
    5. Extension: Find a formula for the minimum number of bounces needed for the rebound height to be under , for a real number .
11. Show that when a quantity is increased by , its new value is . If this quantity is again increased by , what is its new value ? If the operation is performed times in succession, what is the final value of the quantity ?
12. When Eli and Daisy arrive at their cabin in the woods in the middle of winter, the internal temperature is .
    1. Eli wants to turn up the thermostat by every minutes. Find an explicit formula for the sequence that represents the thermostat settings using Eli’s plan.
    2. Daisy wants to turn up the thermostat by every minutes. Find an explicit formula for the sequence that represents the thermostat settings using Daisy’s plan.
    3. Which plan will get the thermostat to most quickly?
    4. Which plan will get the thermostat to most quickly?
13. In nuclear fission, one neutron splits an atom causing the release of two other neutrons, each of which splits an atom and produces the release of two more neutrons, and so on.
    1. Write the first few terms of the sequence showing the numbers of atoms being split at each stage after a single atom splits. Use .
    2. Find the explicit formula that represents your sequence in part (a).
    3. If the interval from one stage to the next is one-millionth of a second, write an expression for the number of atoms being split at the end of one second.
    4. If the number from part (c) were written out, how many digits would it have?