Mathematics Curriculum

Topic D: Using Logarithms in Modeling Situations

A-SSE.B.3c, A-CED.A.1, A-REI.D.11, F-IF.B.3, F-IF.B.6, F-IF.C.8b, F-IF.C.9, F-BF.A.1a, F-BF.1b, F-BF.A.2, F-BF.B.4a, F-LE.A.4, F-LE.B.5

Focus Standards:	A-SSE.B.3c	 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as
		$\left(1.15^{\frac{1}{12}} ight)$ $\approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
	A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
	A-REI.D.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*
	F-IF.B.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for $n \ge 1$.
	F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*



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		F-IF.C.8b	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	
			b. Use the properties of exponents to interpret expressions for exponential functions. <i>For example, identify percent rate of change in functions such</i>	
			as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{\frac{t}{10}}$, and classify them as representing exponential growth or decay.	
		F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i>	
		F-BF.A.1	Write a function that describes a relationship between two quantities. *	
			 Determine an explicit expression, a recursive process, or steps for calculation from a context. 	
			b. Combine standard function types using arithmetic operations. <i>For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</i>	
		F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*	
		F-BF.B.4a	Find inverse functions.	
			a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x + 1)/(x - 1)$ for $x \neq 1$.	
		F-LE.A.4	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.	
		F-LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context.	
Instructiona	al Days:	6		
I	esson 23:	Bean Counting (M) ¹		
I	esson 24:	Solving Exponential Equations (P)		
I	esson 25:	Geometric Sequences and Exponential Growth and Decay (P)		
I	esson 26:	Percent Rate of Change (P)		
I	esson 27:	Modeling with Exponential Functions (M)		
Lesson 28:		Newton's Law of Cooling, Revisited (M)		

¹ Lesson Structure Key: P-Problem Set Lesson, M-Modeling Cycle Lesson, E-Exploration Lesson, S-Socratic Lesson



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This topic opens with a simulation and modeling activity where students start with one bean, roll it out of a cup onto the table, and add more beans each time the marked side is up. While clearly an exponential model, the lesson unfolds by having students discover this relationship, without explicitly stating that the results are exponential, by examining patterns when the data is represented numerically and graphically. Students blend what they know about probability and exponential functions to interpret the parameters *a* and *b* in the functions $f(t) = a(b^t)$ that they find to model their experimental data (**F-LE.B.5**, **A-CED.A.2**).

In both Algebra I and Lesson 6 in this module, students had to solve exponential equations when modeling real-world situations numerically or graphically. Lesson 24 shows students how to use logarithms to solve these types of equations analytically and makes the connections between numeric, graphical, and analytical approaches explicit, invoking the related standards **F-LE.A.4**, **F-BF.B.4a**, and **A-REI.D.11**. Students will be encouraged to use multiple approaches to solve equations generated in the next several lessons.

In Lessons 25 to 27, a general growth/decay rate formula is presented to students to help construct models from data and descriptions of situations. Students must use properties of exponents to rewrite exponential expressions in order to interpret the properties of the function (**F-IF.C.8b**). For example, in Lesson 27, students compare the initial populations and annual growth rates of population functions given in the forms $E(t) = 281.4(1.0093)^{t-100}$, $f(t) = 81.1(1.0126)^t$, and $g(t) = 76.2(13.6)^{t/10}$. Many of the situations and problems presented here were first encountered in Module 3 of Algebra I; students are now able to solve equations involving exponents that they could only estimate previously, such as finding the time when the population of the United States is expected to surpass a half-billion people. Students answer application questions in the context of the situation. They use technology to evaluate logarithms of base 10 and *e*. Additionally, Lesson 25 begins to develop geometric sequences that will be needed for the financial content in the next Topic (**F-BF.A.2**). Lesson 26 continues developing the skills of distinguishing between situations that require exponential-vs-linear models (**F-LE.A.1**), and Lesson 27 continues the work with geometric sequences that started in Lesson 25 (**F-IF.B.3**, **F-BF.A.1a**).

Lesson 28 closes this topic and addresses **F-BF.A.1b** by revisiting Newton's Law of Cooling, a formula that involves the sum of an exponential function and a constant function. Students first learned about this formula in Algebra I but now that they are armed with logarithms and have more experience understanding how transformations affect the graph of a function, they can find the precise value of the decay constant using logarithms and, thus, can solve problems related to this formula more precisely and with greater depth of understanding.



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