Lesson 20: Transformations of the Graphs of Logarithmic and Exponential Functions

Classwork

Opening Exercise

* 1. Sketch the graphs of the three functions , , and .
     1. Describe the transformations that will take the graph of to the graph of .
     2. Describe the transformations that will take the graph of to the graph of .
     3. Explain why and from parts (i) and (ii) are equivalent functions.
  2. Describe the transformations that will take the graph of to the graph of .
  3. Describe the transformations that will take the graph of to the graph of .
  4. Explain why and from parts (b)–(c) are *not* equivalent functions.

Exploratory Challenge

* 1. Sketch the graph of by identifying and plotting at least five key points. Use the table below to help you get started.



* 1. Describe the transformations that will take the graph of to the graph of .
  2. Describe the transformations that will take the graph of to the graph of .
  3. Complete the table below for , , and and describe any patterns that you notice.

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* 1. Graph the three functions on the same coordinate axes and describe any patterns that you notice. Use a property of logarithms to show that and are equivalent.
  2. Describe the graph of as a vertical translation of the graph of . Justify your response.
  3. Describe the graph of as a horizontal scaling of the graph of . Justify your response.
  4. Do the functions and have the same graphs? Justify your reasoning.
  5. Use the properties of exponents to explain why the graphs of and are identical.
  6. Use the properties of exponents to predict what the graphs of and will look like compared to one another. Describe the graphs of and as transformations of the graph of . Confirm your prediction by graphing and on the same coordinate axes.



* 1. Graph , , and on the same coordinate axes. Describe the graphs of and as transformations of the graph of. Use the properties of exponents to explain why and are equivalent.



**Example 1: Graphing Transformations of the Logarithm Functions**

The general form of a logarithm function is given by , where , , , and are real numbers such that is a positive number not equal to and .

* 1. Given , describe the graph of as a transformation of the common logarithm function.
  2. Graph the common logarithm function and on the same coordinate axes.



**Example 2: Graphing Transformations of Exponential Functions**

The general form of the exponential function is given by , where , , and are real numbers such that is a positive number not equal to .

* 1. Use the properties of exponents to transform the function to the general form, and then graph it. What are the values of , , and ?
  2. Describe the graph of as a transformation of the graph of .
  3. Describe the graph of as a transformation of the graph of .
  4. Graph using transformations.



Exercises 1–4

Graph each pair of functions by first graphing and then graphing by applying transformations of the graph of . Describe the graph of as a transformation of the graph of .

1. and
2. and
3. and
4. and

Problem Set

Lesson Summary

**General form of a logarithmic function:** such that , , and are real numbers, is any positive number not equal to , and .

**General form of an exponential function:** such that and are real numbers, and is any positive number not equal to .

The properties of logarithms and exponents can be used to rewrite expressions for functions in equivalent forms that can then be graphed by applying transformations.

1. Describe each function as a transformation of the graph of a function in the form . Sketch the graph of and the graph of by hand. Label key features such as intercepts, increasing or decreasing intervals, and the equation of the vertical asymptote.
2. Each function graphed below can be expressed as a transformation of the graph of . Write an algebraic function for and and state the domain and range.



Figure : Graphs of and the function Figure : Graphs of and the function

1. Describe each function as a transformation of the graph of a function in the form . Sketch the graph of and the graph of by hand. Label key features such as intercepts, increasing or decreasing intervals, and the horizontal asymptote. (Estimate when needed from the graph.)
2. Using the function , create a new function whose graph is a series of transformations of the graph of with the following characteristics:

* The graph of is decreasing for all real numbers.
* The equation for the horizontal asymptote is .
* The -intercept is .

1. Using the function , create a new function whose graph is a series of transformations of the graph of with the following characteristics:

* The graph of is increasing for all real numbers.
* The equation for the horizontal asymptote is .
* The -intercept is .

1. Given the function :
   1. Write the function as an exponential function with base . Describe the transformations that would take the graph of to the graph of .
   2. Write the function as an exponential function with base . Describe two different series of transformations that would take the graph of to the graph of .
2. Explore the graphs of functions in the form for . Explain how the graphs of these functions change as the values of increase. Use a property of logarithms to support your reasoning.
3. Use a graphical approach to solve each equation. If the equation has no solution, explain why.
   1. Show algebraically that the exact solution to the equation in part (c) is .
4. Make a table of values for for . Graph this function for . Use properties of logarithms to explain what you see in the graph and the table of values.