Lesson 19: The Inverse Relationship Between Logarithmic and Exponential Functions

Classwork

Opening Exercise

* 1. Consider the mapping diagram of the function below. Fill in the blanks of the mapping diagram of to construct a function that “undoes” each output value of by returning the original input value of . (The first one is done for you.)

Domain Range

\_\_\_  
\_\_\_  
\_\_\_  
\_\_\_

Domain Range

* 1. Write the set of input-output pairs for the functions and by filling in the blanks below. (The set for the function has been done for you.)  
        
       
       
      \_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_
  2. How can the points in the set be obtained from the points in ?
  3. Peter studied the mapping diagrams of the functions and above and exclaimed, “I can get the mapping diagram for by simply taking the mapping diagram for and reversing all of the arrows!” Is he correct?

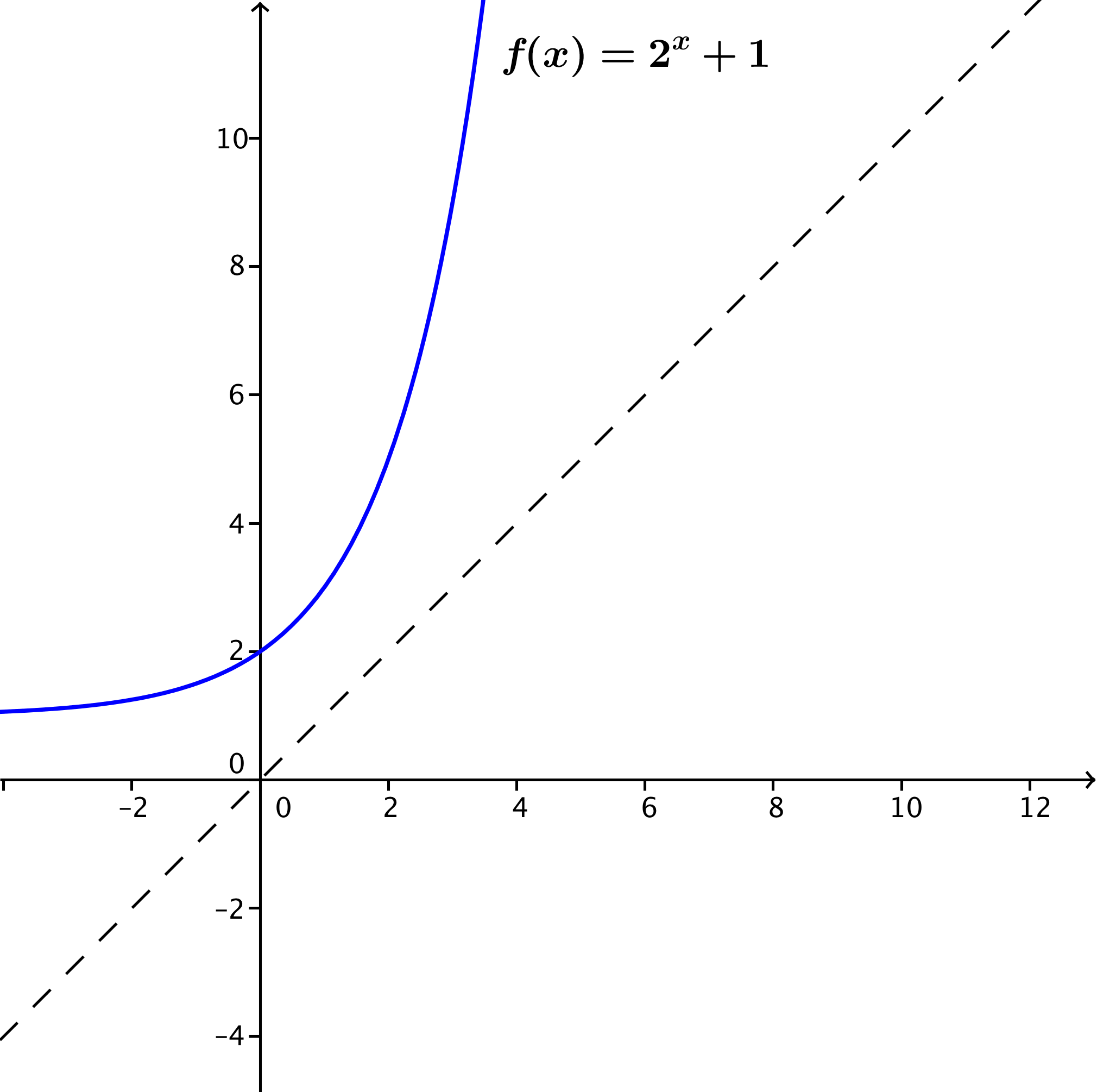
Exercises

For each function in Exercises 1–5, find the formula for the corresponding inverse function . Graph both functions on a calculator to check your work.

1. for
2. for
3. Cindy thinks that the inverse of is . To justify her answer, she calculates and then substitutes the output into to get , which gives back the original input. Show that Cindy is incorrect by using other examples from the domain and range of .
4. After finding the inverse for several functions, Henry claims that every function must have an inverse. Rihanna says that his statement is not true and came up with the following example: If has an inverse, then because and both have the same output , the inverse function would have to map to both and simultaneously, which violates the definition of a function. What is another example of a function without an inverse?

**Example**

Consider the function , whose graph is shown at right.

* 1. What are the domain and range of ?
  2. Sketch the graph of the inverse function on the graph. What type of function do you expect to be?
  3. What are the domain and range of ? How does that relate to your answer in part (a)?
  4. Find the formula for .

Lesson Summary

* **Invertible Function:** Let be a function whose domain is the set and whose image is the set . Then is *invertible* if there exists a function with domain and image such that and satisfy the property:

For all in and in if and only if .

* The function is called the *inverse* of .
* If two functions whose domain and range are a subset of the real numbers are inverses, then their graphs are reflections of each other across the diagonal line given by in the Cartesian plane.
* If and are inverses of each other, then
  + The domain of is the same set as the range of .
  + The range of is the same set as the domain of .
* In general, to find the formula for an inverse function of a given function :
  + Write using the formula for .
  + Interchange the symbols and to get .
  + Solve the equation for to write as an expression in .
  + Then, the formula for is the expression in found in step (iii).
* The functions and are inverses of each other.

Problem Set

1. For each function below, find two functions and such that . (There are many correct answers.)
2. Let be the function that assigns to each student in your class his or her biological mother.
   1. Use the definition of function to explain why is a function.
   2. In order for to have an inverse, what condition must be true about the students in your class?
   3. If we enlarged the domain to include all students in your school, would this larger domain function have an inverse?
3. The table below shows a partially filled-out set of input-output pairs for two functions and that have the same finite domain of .

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |

* 1. Complete the table so that is invertible but is definitely not invertible.
  2. Graph both functions and use their graphs to explain why is invertible and is not.

1. Find the inverse of each of the following functions. In each case, indicate the domain and range of both the original function and its inverse.
2. Unlike square roots that do not have any real principal square roots for negative numbers, principal cube roots do exist for negative numbers: is the real number since it satisfies . Use the identities  
    and for any real number to find the inverse of each of the functions below. In each case, indicate the domain and range of both the original function and its inverse.
   1. for any real number .
   2. for any real number .
   3. for any real number .
3. Suppose that the inverse of a function is the function itself. For example, the inverse of the function (for ) is just itself again, (for ). What symmetry must the graphs of all such functions have?   
   (Hint: Study the graph of Exercise 5 in the lesson.)
4. When traveling abroad, you will find that daily temperatures in other countries are often reported in Celsius. The sentence, “It will be today in Paris,” does not mean it will be freezing in Paris. It will often be necessary for you to convert temperatures reported in degrees Celsius to degrees Fahrenheit, the scale we use in the U.S. for reporting daily temperatures.

Let *f* be the function that inputs a temperature measure in degrees Celsius and outputs the corresponding temperature measure in degrees Fahrenheit.

* 1. Assuming that is linear, we can use two points on the graph of to determine a formula for . In degrees Celsius, the freezing point of water is and its boiling point is . In degrees Fahrenheit, the freezing point of water is and its boiling point is . Use this information to find a formula for the function .   
     (Hint: Plot the points and draw the graph of first, keeping careful track of the meaning of values on the   
     -axis and -axis.)
  2. What temperature will Paris be in degrees Fahrenheit if it is reported that it will be ?
  3. Find the inverse of the function and explain its meaning in terms of degree scales that its domain and range represent.
  4. The graphs of and its inverse are two lines that intersect in one point. What is that point? What is its significance in terms of degrees Celsius and degrees Fahrenheit?

**Extension:** Use the fact that, for , the functions and are increasing to solve the following problems. Recall that an increasing function has the property that if both and are in the domain of and , then .

1. For which values of is ?
2. For which values of is ?