Lesson 17: Graphing the Logarithm Function

Classwork

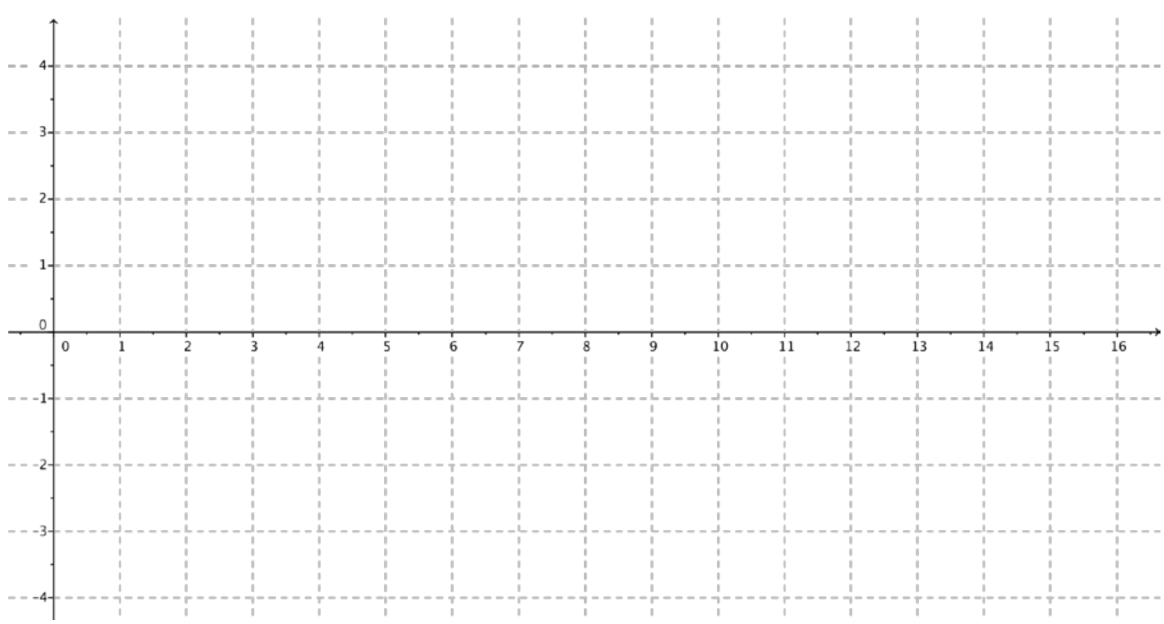
Opening Exercise

Graph the points in the table for your assigned function $f(x) = \log(x)$, $g(x) = \log_2(x)$, or $h(x) = \log_5(x)$ for $0 < x \leq 16$. Then, sketch a smooth curve through those points and answer the questions that follow.

10-team $f(x) = \log(x)$	
x	$f(x)$
0.0625	-1.20
0.125	-0.90
0.25	-0.60
0.5	-0.30
1	0
2	0.30
4	0.60
8	0.90
16	1.20

2-team $g(x) = \log_2(x)$	
x	$g(x)$
0.0625	-4
0.125	-3
0.25	-2
0.5	-1
1	0
2	1
4	2
8	3
16	4

5-team $h(x) = \log_5(x)$	
x	$h(x)$
0.0625	-1.72
0.125	-1.29
0.25	-0.86
0.5	-0.43
1	0
2	0.43
4	0.86
8	1.29
16	1.72



a. What does the graph indicate about the domain of your function?

b. Describe the x-intercepts of the graph.

c. Describe the y-intercepts of the graph.

d. Find the coordinates of the point on the graph with y-value 1.

e. Describe the behavior of the function as $x \to 0$.

f. Describe the end behavior of the function as $x \to \infty$.

g. Describe the range of your function.

h. Does this function have any relative maxima or minima? Explain how you know.

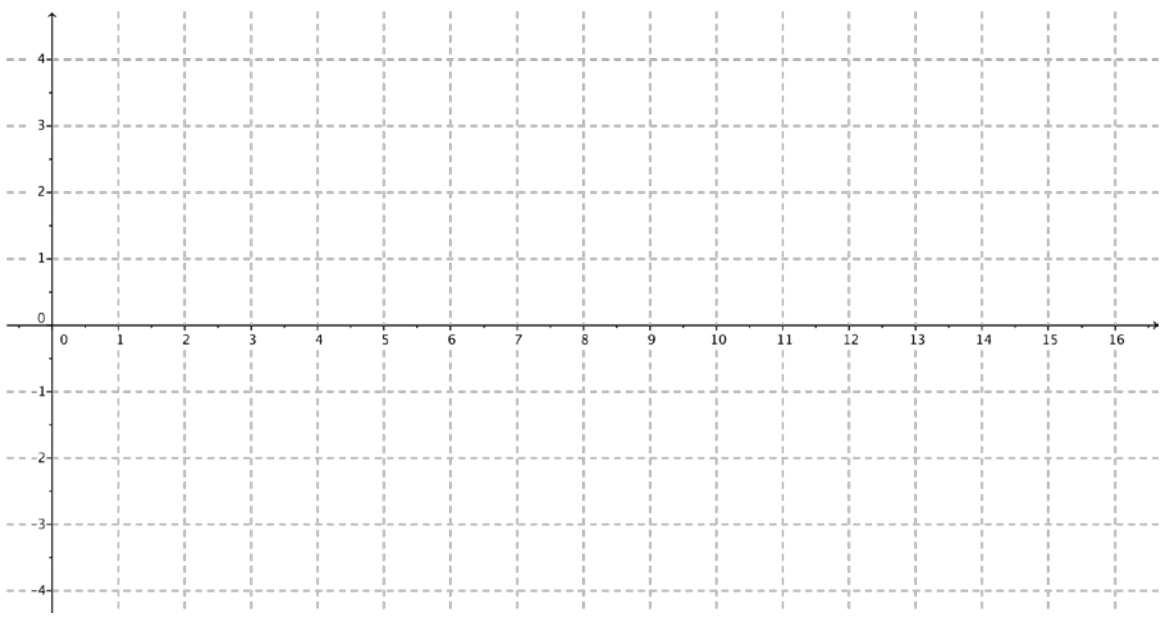
Exercises

1. Graph the points in the table for your assigned function $r(x) = \log_{\frac{1}{10}}(x)$, $s(x) = \log_{\frac{1}{2}}(x)$, or $t(x) = \log_{\frac{1}{5}}(x)$ for $0 < x \leq 16$. Then sketch a smooth curve through those points, and answer the questions that follow.

10-team $r(x) = \log_{\frac{1}{10}}(x)$	
x	$r(x)$
0.0625	1.20
0.125	0.90
0.25	0.60
0.5	0.30
1	0
2	-0.30
4	-0.60
8	-0.90
16	-1.20

2-team $s(x) = \log_{\frac{1}{2}}(x)$	
x	$s(x)$
0.0625	4
0.125	3
0.25	2
0.5	1
1	0
2	-1
4	-2
8	-3
16	-4

e-team $t(x) = \log_{\frac{1}{5}}(x)$	
x	$t(x)$
0.0625	1.72
0.125	1.29
0.25	0.86
0.5	0.43
1	0
2	-0.43
4	-0.86
8	-1.29
16	-1.72



a. What is the relationship between your graph in the Opening Exercise and your graph from this exercise?

b. Why does this happen? Use the change of base formula to justify what you have observed in part (a).

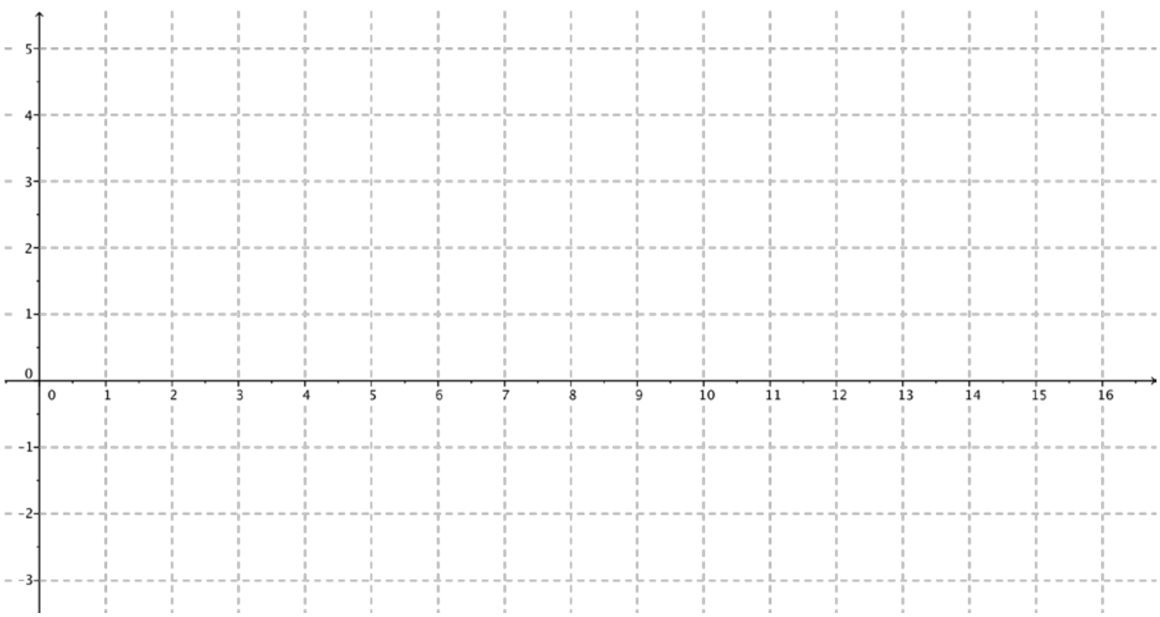
2. In general, what is the relationship between the graph of a function $y = f(x)$ and the graph of $y = f(kx)$ for a constant k?

3. Graph the points in the table for your assigned function $u(x) = \log(10x)$, $v(x) = \log_2(2x)$, or $w(x) = \log_5(5x)$ for $0 < x \leq 16$. Then sketch a smooth curve through those points, and answer the questions that follow.

10-team $u(x) = \log(10x)$	
x	$u(x)$
0.0625	-0.20
0.125	0.10
0.25	0.40
0.5	0.70
1	1
2	1.30
4	1.60
8	1.90
16	2.20

2-team $v(x) = \log_2(2x)$	
x	$v(x)$
0.0625	-3
0.125	-2
0.25	-1
0.5	0
1	1
2	2
4	3
8	4
16	5

5-team $w(x) = \log_5(5x)$	
x	$w(x)$
0.0625	-0.72
0.125	-0.29
0.25	0.14
0.5	0.57
1	1
2	1.43
4	1.86
8	2.29
16	2.72



a. Describe a transformation that takes the graph of your team's function in this exercise to the graph of your team's function in the Opening Exercise.

b. Do your answers to Exercise 2 and part (a) agree? If not, use properties of logarithms to justify your observations in part (a).

Lesson Summary

The function $f(x) = \log_b(x)$ is defined for irrational and rational numbers. Its domain is all positive real numbers. Its range is all real numbers.

The function $f(x) = \log_b(x)$ goes to negative infinity as x goes to zero. It goes to positive infinity as x goes to positive infinity.

The larger the base b, the more slowly the function $f(x) = \log_b(x)$ increases.

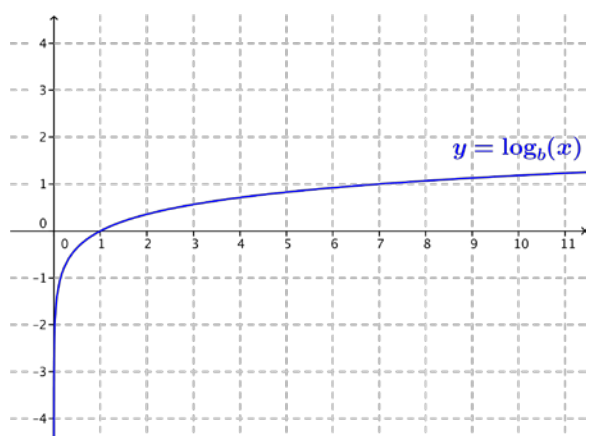
By the change of base formula, $\log_{\frac{1}{b}}(x) = -\log_b(x)$.

Problem Set

1. The function $Q(x) = \log_b(x)$ has function values in the table at right.
   a. Use the values in the table to sketch the graph of $y = Q(x)$.
   b. What is the value of b in $Q(x) = \log_b(x)$? Explain how you know.
   c. Identify the key features in the graph of $y = Q(x)$.

x	$Q(x)$
0.1	1.66
0.3	0.87
0.5	0.50
1.00	0.00
2.00	-0.50
4.00	-1.00
6.00	-1.29
10.00	-1.66
12.00	-1.79

2. Consider the logarithmic functions $f(x) = \log_b(x)$, $g(x) = \log_5(x)$, where b is a positive real number, and $b \neq 1$. The graph of f is given at right.
   a. Is $b > 5$, or is $b < 5$? Explain how you know.
   b. Compare the domain and range of functions f and g.
   c. Compare the x-intercepts and y-intercepts of f and g.
   d. Compare the end behavior of f and g.



3. Consider the logarithmic functions $f(x) = \log_b(x)$, $g(x) = \log_{\frac{1}{2}}(x)$, where b is a positive real number and $b \neq 1$. A table of approximate values of f is given below.

x	$f(x)$
$\frac{1}{4}$	0.86
$\frac{1}{2}$	0.43
1	0
2	-0.43
4	-0.86

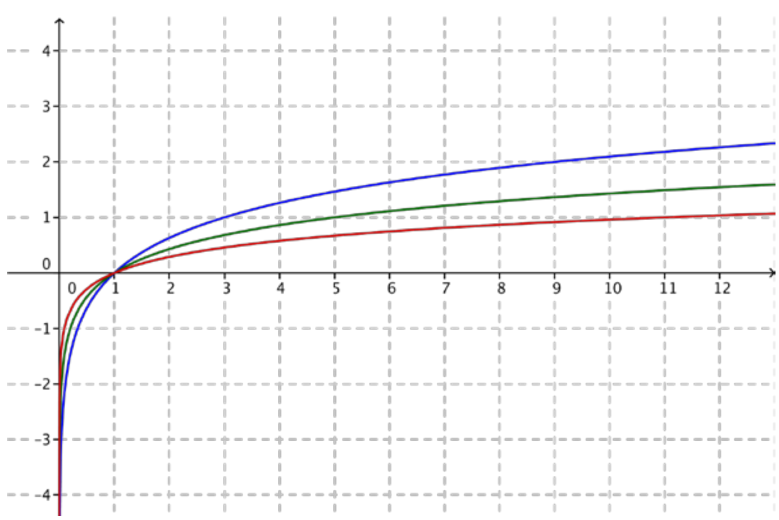
   a. Is $b > \frac{1}{2}$, or is $b < \frac{1}{2}$? Explain how you know.
   b. Compare the domain and range of functions f and g.
   c. Compare the x-intercepts and y-intercepts of f and g.
   d. Compare the end behavior of f and g.

4. On the same set of axes, sketch the functions $f(x) = \log_2(x)$ and $g(x) = \log_2(x^3)$.
   a. Describe a transformation that takes the graph of f to the graph of g.
   b. Use properties of logarithms to justify your observations in part (a).

5. On the same set of axes, sketch the functions $f(x) = \log_2(x)$ and $g(x) = \log_2\left(\frac{x}{4}\right)$.
   a. Describe a transformation that takes the graph of f to the graph of g.
   b. Use properties of logarithms to justify your observations in part (a).

6. On the same set of axes, sketch the functions $f(x) = \log_{\frac{1}{2}}(x)$ and $g(x) = \log_2\left(\frac{1}{x}\right)$.
   a. Describe a transformation that takes the graph of f to the graph of g.
   b. Use properties of logarithms to justify your observations in part (a).

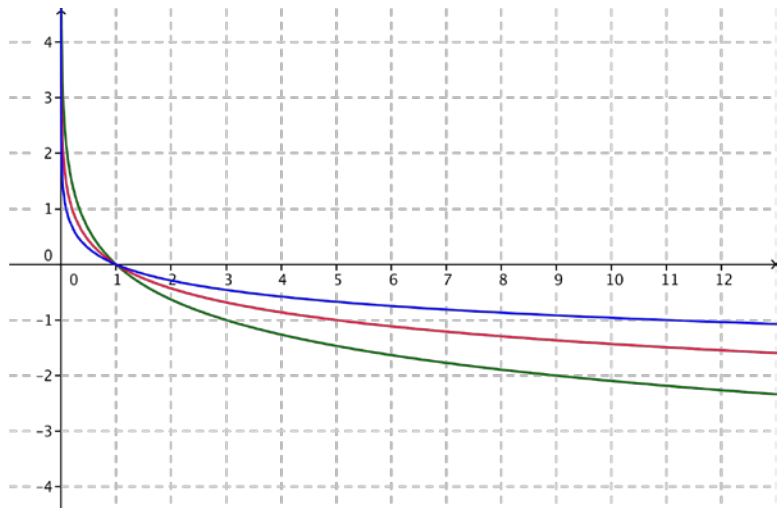
7. The figure below shows graphs of the functions $f(x) = \log_3(x)$, $g(x) = \log_5(x)$, and $h(x) = \log_{11}(x)$.
   a. Identify which graph corresponds to which function. Explain how you know.
   b. Sketch the graph of $k(x) = \log_7(x)$ on the same axes.



8. The figure below shows graphs of the functions $f(x) = \log_{\frac{1}{3}}(x)$, $g(x) = \log_{\frac{1}{5}}(x)$, and $h(x) = \log_{\frac{1}{11}}(x)$.

   a. Identify which graph corresponds to which function. Explain how you know.

   b. Sketch the graph of $k(x) = \log_{\frac{1}{7}}(x)$ on the same axes.



9. For each function f, find a formula for the function h in terms of x. Part (a) has been done for you.

   a. If $f(x) = x^2 + x$, find $h(x) = f(x+1)$.

   b. If $f(x) = \sqrt{x^2 + \frac{1}{4}}$, find $h(x) = f\left(\frac{1}{2}x\right)$.

   c. If $f(x) = \log(x)$, find $h(x) = f\left(\sqrt[3]{10x}\right)$ when $x > 0$.

   d. If $f(x) = 3^x$, find $h(x) = f(\log_3(x^2+3))$.

   e. If $f(x) = x^3$, find $h(x) = f\left(\frac{1}{x^3}\right)$ when $x \neq 0$.

   f. If $f(x) = x^3$, find $h(x) = f\left(\sqrt[3]{x}\right)$.

   g. If $f(x) = \sin(x)$, find $h(x) = f\left(x + \frac{\pi}{2}\right)$.

   h. If $f(x) = x^2 + 2x + 2$, find $h(x) = f(\cos(x))$.

10. For each of the functions f and g below, write an expression for (i) $f\big(g(x)\big)$, (ii) $g\big(f(x)\big)$, and (iii) $f\big(f(x)\big)$ in terms of x. Part (a) has been done for you.

    a. $f(x) = x^2$, $g(x) = x + 1$

        i. $f\big(g(x)\big) = f(x+1)$
        $= (x+1)^2$

        ii. $g\big(f(x)\big) = g(x^2)$
        $= x^2 + 1$

        iii. $f\big(f(x)\big) = f(x^2)$
        $= (x^2)^2$
        $= x^4$

    b. $f(x) = \frac{1}{4}x - 8$, $g(x) = 4x + 1$

    c. $f(x) = \sqrt[3]{x+1}$, $g(x) = x^3 - 1$

    d. $f(x) = x^3$, $g(x) = \frac{1}{x}$

    e. $f(x) = |x|$, $g(x) = x^2$

Extension:

11. Consider the functions $f(x) = \log_2(x)$ and $(x) = \sqrt{x-1}$.

    a. Use a calculator or other graphing utility to produce graphs of $f(x) = \log_2(x)$ and $g(x) = \sqrt{x-1}$ for $x \leq 17$.

    b. Compare the graph of the function $f(x) = \log_2(x)$ with the graph of the function $g(x) = \sqrt{x-1}$. Describe the similarities and differences between the graphs.

    c. Is it always the case that $\log_2(x) > \sqrt{x-1}$ for $x > 2$?

12. Consider the functions $f(x) = \log_2(x)$ and $(x) = \sqrt[3]{x-1}$.

    a. Use a calculator or other graphing utility to produce graphs of $f(x) = \log_2(x)$ and $h(x) = \sqrt[3]{x-1}$ for $x \leq 28$.

    b. Compare the graph of the function $f(x) = \log_2(x)$ with the graph of the function $h(x) = \sqrt[3]{x-1}$. Describe the similarities and differences between the graphs.

    c. Is it always the case that $\log_2(x) > \sqrt[3]{x-1}$ for $x > 2$?