



Topic C:

Exponential and Logarithmic Functions and their Graphs

F-IF.B.4, F-IF.B.5, F-IF.C.7e, F-BF.A.1a, F-BF.B.3, F-BF.B.4a, F-LE.A.2, F-LE.A.4

Focus Standards:	F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*</i>
	F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*</i>
	F-IF.C.7e	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
	F-BF.A.1a	Write a function that describes a relationship between two quantities.* a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
	F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. <i>Include recognizing even and odd functions from their graphs and algebraic expressions for them.</i>

- F-BF.B.4a Find inverse functions.
- Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ or $f(x) = (x + 1)/(x - 1)$ for $x \neq 1$.*
- F-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- F-LE.A.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a , c , and d are numbers, and the base b is 2, 10, or e ; evaluate the logarithm using technology.

Instructional Days: 7

Lesson 16: Rational and Irrational Numbers (S)¹

Lesson 17: Graphing the Logarithm Function (P)

Lesson 18: Graphs of Exponential Functions and Logarithmic Functions (P)

Lesson 19: The Inverse Relationship between Logarithmic and Exponential Functions (P)

Lesson 20: Transformations of the Graphs of Logarithmic and Exponential Functions (E)

Lesson 21: The Graph of the Natural Logarithm Function (E)

Lesson 22: Choosing a Model (P)

The lessons covered in Topic A and Topic B build upon students' prior knowledge of the properties of exponents, exponential expression, and solving equations by extending the properties of exponents to all real number exponents and positive real number bases before introducing logarithms. This topic reintroduces exponential functions, introduces logarithmic functions, explains their inverse relationship, and explores the features of their graphs and how they can be used to model data.

Lesson 16 ties back to work in Topic A by helping students to further extend their understanding of the properties of real numbers, both rational and irrational (**N-RN.B.3**). This Algebra I standard is revisited in Algebra II so that students know and understand that the exponential functions are defined for all real numbers, and, thus, the graphs of the exponential functions can be represented by a smooth curve. Another consequence is that the logarithm functions are also defined for all positive real numbers. Lessons 17 and 18 introduce the graphs of logarithmic functions and exponential functions. Students compare the properties of graphs of logarithm functions for different bases and identify common features, which align with standards **F-IF.B.4**, **F-IF.B.5**, and **F-IF.C.7**. Students understand that if the range of this function is all real numbers, then some logarithms must be irrational. Students notice that $f(x) = b^x$ and $g(x) = \log_b(x)$ appear to be related via a reflection across the graph of the equation $y = x$.

Lesson 19 addresses standards **F-BF.B.4a** and **F-LE.A.4** while continuing the ideas introduced graphically in Lesson 18 to help students make the connection that the logarithmic function and the exponential function are inverses of each other. Inverses are introduced first by discussing operations and functions that can “undo” each other; then, students look at the graphs of pairs of these functions. The lesson ties the ideas back to reflections in the plane from geometry and illuminates why the graphs of inverse functions are

¹ Lesson Structure Key: **P**-Problem Set Lesson, **M**-Modeling Cycle Lesson, **E**-Exploration Lesson, **S**-Socratic Lesson

reflections of each other across the line given by $y = x$, developing these ideas intuitively without formalizing what it means for two functions to be inverses. Inverse functions will be addressed in greater detail in Precalculus.

During all of these lessons, connections are made to the properties of logarithms and exponents. The relationship between graphs of these functions, applying transformations to sketch graphs of functions, and the properties associated with these functions are linked in Lessons 20 and 21, showcasing standards **F-IF.C.7e** and **F-BF.B.3**. Students use properties and their knowledge of transformations to explain why two different functions such as $f(x) = \log(10x)$ and $g(x) = 1 + \log(x)$ have the same graph. Lesson 21 revisits the natural logarithm function, and students see how the change of base property of logarithms proves that we can write a logarithm function of any base b as a vertical scaling of the natural logarithm function (or any other base logarithm function we choose).

Finally, in Lesson 22, students must synthesize knowledge across both Algebra I and Algebra II to decide whether a linear, quadratic, sinusoidal, or exponential function will best model a real-world scenario by analyzing the way in which we expect the quantity in question to change. For example, students need to determine whether or not to model daylight hours in Oslo, Norway, with a linear or a sinusoidal function because the data appears to be linear, but that does not make sense in context. They model the outbreak of a flu epidemic with an exponential function and a falling body with a quadratic function. In this lesson, the majority of the scenarios that require modeling are described verbally, and students determine an explicit expression for many of the functions in accordance with **F-BF.A.1a**, **F-LE.A.1**, and **F-LE.A.2**.