Lesson 16: Rational and Irrational Numbers

Classwork

Opening Exercise

* 1. Explain how to use a number line to add the fractions
  2. Convert and to decimals, and explain the process for adding them together.

Exercises

1. According to the calculator, ... and .... Find an approximation of to one decimal place, that is, to an accuracy of .
2. Find the value of to an accuracy of .
3. Find the value of to an accuracy of .
4. Make a conjecture: Is a rational or an irrational number?
5. Why is your conjecture in Exercise 4 true?

Remember that the calculator gives the following values: … and ….

1. Find the value of to three decimal places.
2. Find the value of to five decimal places.
3. Does your conjecture from the above discussion appear to be true?

Lesson Summary

* Irrational numbers occur naturally and frequently.
* The  th roots of most integers and rational numbers are irrational.
* Logarithms of most positive integers or positive rational numbers are irrational.
* We can locate an irrational number on the number line by trapping it between lower and upper approximations. The infinite process of squeezing the irrational number in smaller and smaller intervals locates exactly where the irrational number is on the number line.
* We can perform arithmetic operations such as addition and multiplication with irrational numbers using lower and upper approximations and squeezing the result of the operation in smaller and smaller intervals between two rational approximations to the result.

Problem Set

1. Given that and , find the sum to an accuracy of , without using a calculator.
2. Put the following numbers in order from least to greatest.
3. Find a rational number between the specified two numbers.
   1. and
   2. and
   3. and
   4. and
   5. and
4. Knowing that is irrational, find an irrational number between and .
5. Give an example of an irrational number between and .
6. Given that is irrational, which of the following numbers are irrational?
7. Given that is irrational, which of the following numbers are irrational?
8. Which of the following numbers are irrational?
9. Find two irrational numbers and so that their average is rational.
10. Suppose that is an irrational number. Explain how you know that must be an irrational number. (Hint: What would happen if there were integers and so that ?)
11. If and are rational numbers, prove that and are also rational numbers.
12. If is a rational number and is an irrational number, determine whether the following numbers are always rational, sometimes rational, or never rational. Explain how you know.
13. If and are irrational numbers, determine whether the following numbers are always rational, sometimes rational, or never rational. Explain how you know.