Lesson 13: Changing the Base

Classwork

Exercises

1. Assume that ,, and are all positive real numbers, so that and . What is in terms of ? The resulting equation allows us to change the base of a logarithm from to .
2. Approximate each of the following logarithms to four decimal places. Use the log key on your calculator rather than logarithm tables, first changing the base of the logarithm to if necessary.
3. In Lesson 12, we justified a number of properties of base logarithms. Working in pairs, justify the following properties of base logarithms.
4. Find each of the following to four decimal places. Use the ln key on your calculator rather than a table.

1. Write as a single logarithm:
   1. .
   2. .
2. Write each expression as a sum or difference of constants and logarithms of simpler terms.

Lesson Summary

We have established a formula for changing the base of logarithms from to :

In particular, the formula allows us to change logarithms base to common or natural logarithms, which are the only two kinds of logarithms that calculators compute:

We have also established the following properties for base logarithms. If ,,, and are all positive real numbers with and and is any real number, then:

Problem Set

1. Evaluate each of the following logarithmic expressions, approximating to four decimal places if necessary. Use the  
    ln or log key on your calculator rather than a table.
2. Use logarithmic properties and the fact that and to approximate the value of each of the following logarithmic expressions. Do not use a calculator.
3. Compare the values of and without using a calculator.
4. Show that for any positive numbers and with and , .
5. Express in terms of ,, and if .
6. Rewrite each expression in an equivalent form that only contains one base logarithm.
   1. , for positive real values of
7. Write each number in terms of natural logarithms, and then use the properties of logarithms to show that it is a rational number.
8. Write each expression as an equivalent expression with a single logarithm. Assume , , and are positive real numbers.
9. Rewrite each expression as sums and differences in terms of , , and .
10. Solve the following equations in terms of base logarithms. Then, use the change of base properties and a calculator to estimate the solution to the nearest th. If the equation has no solution, explain why.
11. In Lesson 6, you discovered that by looking at a table of logarithms. Use the properties of logarithms to justify this property for an arbitrary base with . That is, show that
12. Larissa argued that since and , then it must be true that . Is she correct? Explain how you know.
13. Extension: Suppose that there is some positive number so that
    1. Use the given values of , , and to evaluate the following logarithms.
    2. Use the change of base formula to convert to base and solve for . Give your answer to four decimal places.
14. Solve the following exponential equations.
15. Solve each exponential equation.

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1. In Problem 9(e) of Lesson 12, you solved the equation using the logarithm base .
   1. Solve using the logarithm base .
   2. Apply the change of base formula to show that your answer to part (a) agrees with your answer to Problem 9(e) of Lesson 12.
   3. Solve using the logarithm base .
   4. Apply the change of base formula to show that your answer to part (c) also agrees with your answer to Problem9(e) of Lesson 12.
2. Pearl solved the equation as follows:

Jess solved the equation as follows:

Is Pearl correct? Is Jess correct? Explain how you know.