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Lesson 10: Building Logarithmic Tables

Student Outcomes

* Students construct a table of logarithms base and observe patterns that indicate properties of logarithms.

Lesson Notes

In the previous lesson, students were introduced to the concept of the logarithm by finding the power to which we need to raise a base in order to produce a given number, which we originally called the function. In this lesson and the next, students will build their own base logarithm tables using their calculators. By taking the time to construct the table themselves (as opposed to being handed a pre-prepared table), students will have a better opportunity to observe patterns in the table and practice MP.7. These observed patterns will lead to formal statements of the properties of logarithms in upcoming lessons. Using logarithmic properties to rewrite logarithmic expressions satisfies the foundational standard **A-SSE.A.2**.

Materials Needed

Students will need access to a calculator or other technological tool able to compute exponents and logarithms base .

Classwork

Opening Exercise (3 minutes)

In this quick Opening Exercise, we ask students to recall the function from the previous lesson and reinforce that the logarithm base is the formal name of the function. We will consider only base logarithms in this lesson as we construct our table, so this Opening Exercise is constrained to base calculations.

*Scaffolding:*

Prompt struggling students to restate the logarithmic equation as the exponential equation .

At the end of this exercise, announce to the students that the notation without the little in the subscript means . This is called the *common logarithm.*

Opening Exercise

Find the value of the following expressions without using a calculator.

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Formulate a rule based on your results above: If is an integer, then .

Example 1 (6 minutes)

In this example, we get our first glimpse of the property Be careful not to give this formula away; by the end of the next lesson, students should have discovered it for themselves.

* Suppose that you are an astronomer, and you measure the distance to a star as miles. A second star is collinear with the first star and the Earth and is times farther away from Earth than the first star is. How many miles is the second star from Earth? Note: The figure is not to scale.

Example 1



100,000,000,000,000 Miles

1,000,000 times farther away

* + *, so the second star is quintillion miles away from Earth.*
* How did you arrive at that figure?
  + *I counted the zeros; there are zeros in and zeros in, so there must be zeros in the product.*
* Can we restate that in terms of exponents?
* How are the exponents related?
* What are , , and ?
  + , , *and*
* In this case, can we state an equivalent expression for ?
* Why is this equation true?
* Generalize to find an equivalent expression for for integers and . Why is this equation true?

**MP.8**

* + *This equation is true because when we multiply powers of together, the resulting product is a power of whose exponent is the sum of the exponents of the factors.*
* Keep this result in mind as we progress through the lesson.

Exercises 1–6 (8 minutes)

Historically, logarithms were calculated using tables because there were no calculators or computers to do the work. Every scientist and mathematician kept a book of logarithmic tables on hand to use for calculation. It is very easy to find the value of a base logarithm for a number that is a power of , but what about for the other numbers? In this exercise, students will find an approximate value of using exponentiation, the same way we approximated in Lesson 6. After this exercise, we will rely on the logarithm button on the calculator to compute logarithms base for the remainder of this lesson. Emphasize to the students that logarithms are generally irrational numbers, so that the results produced by the calculator are only decimal approximations. As such, we should be careful to use the approximation symbol, , when writing out a decimal expansion of a logarithm.

Exercises

1. Find two consecutive powers of so that is between them. That is, find an integer exponent so that   
   .

Since , we have .

1. From your result in Exercise 1, is between which two integers?

***Since is some power of between and , .***

1. Find a number to one decimal place so that , and use that to find under and over estimates for .

***Since and , we have . Then , and***

1. Find a number to two decimal places so that , and use that to find under and over estimates for .

*Since , and , we have so that . So, .*

1. Repeat this process to approximate the value of to decimal places.

***Since , and , we have so that   
.***

***Since , and , we have so that   
.***

***Since , and , we have so that***

***Thus, to four decimal places, .***

1. Verify your result on your calculator, using the log button.

***The calculator gives .***

**Discussion (1 minute)**

In the next exercises, students will use their calculators to create a table of logarithms that they will analyze to look for patterns that will lead to the discovery of the logarithmic properties. The process of identifying and generalizing the observed patterns provides students with an opportunity to practice MP.7.

* Historically, since there were no calculators or computers, logarithms were calculated using a complicated algorithm involving multiple square roots. Thankfully, we have calculators and computers to do this work for us now.
* We will use our calculators to create a table of values of base logarithms. Once the table is made, see what patterns you can observe.

Exercises 7–10 (6 minutes)

Put students in pairs or small groups, but have students work individually to complete the table in Exercise 7. Before working on Exercises 8–10 in groups, have students check their tables against each other. You may need to remind students that means .

1. Use your calculator to complete the following table. Round the logarithms to 4 decimal places.

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1. What pattern(s) can you see in the table from Exercise 7 as is multiplied by ? Write the pattern(s) using logarithmic notation.

***I found the patterns and . I also noticed that   
.***

**MP.7**

1. What pattern would you expect to find for ? Make a conjecture and test it to see whether or not it appears to be valid.

***I would guess that the values of will all start with . That is, . This appears to be the case since , , and .***

1. Use your results from Exercises 8 and 9 to make a conjecture about the value of for any positive integer .

***It appears that , for any positive integer .***

**Discussion (3 minutes)**

Ask groups to share the patterns they observed in Exercise 8 and the conjectures they made in Exercises 9 and 10. Ensure that all students have the correct conjectures recorded in their notebooks or journals before continuing to the next set of exercises, which extend the result from Exercise 10 to all integers (and not just positive values of ).

Exercises 11–14 (8 minutes)

*Scaffolding:*

If students are having difficulty seeing the pattern in the table for Exercise 12, nudge them to add together and for some values of in the table.

In this set of exercises, students will discover a rule for calculating logarithms of the form , where is any integer. Have students again work individually to complete the table in Exercise 11 and to check their tables against each other before they proceed to discuss and answer Exercises 12–14 in groups.

1. Use your calculator to complete the following table. Round the logarithms to decimal places.

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1. What pattern(s) can you see in the table from Exercise ? Write them using logarithmic notation.

**MP.7**

***I found the patterns , which can be written as , and   
.***

1. What pattern would you expect to find for ? Make a conjecture, and test it to see whether or not it appears to be valid.

***I would guess that the values of will all start with , and that . This appears to be the case since , and ; , and ; , and .***

**MP.7**

1. Combine your results from Exercises 10 and 12 to make a conjecture about the value of the logarithm for a multiple of a power of ; that is, find a formula for for any integer .

***It appears that , for any integer .***

**Discussion (2 minutes)**

Ask groups to share the patterns they observed in Exercise 12 and the conjectures they made in Exercises 13 and 14 with the class. Ensure that all students have the correct conjectures recorded in their notebooks or journals before continuing to the next example.

Examples 2–3 (2 minutes)

Lead the class through these calculations. You may decide to let them work on Example 3 either alone or in groups after you have led them through Example 2.

Example 2

Use the logarithm tables and the rules we discovered to calculate to decimal places.

Example 3

Use the logarithm tables and the rules we discovered to calculate to decimal places.

Closing (2 minutes)

Ask students to summarize the important parts of the lesson, either in writing, to a partner, or as a class. Use this as an opportunity to informally assess understanding of the lesson. The following are some important summary elements:

Lesson Summary

* **The notation is used to represent .**
* **For integers , .**
* **For integers and , .**
* **For integers and positive real numbers, .**

Exit Ticket (4 minutes)

Name Date

Lesson 10: Building Logarithmic Tables

Exit Ticket

1. Use the log table below to approximate the following logarithms to four decimal places. Do not use a calculator.

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1. Suppose that is a number with .
   1. What is the value of ?
   2. Which of the following is true? Explain how you know.

Exit Ticket Sample Solutions

1. Use the log table below to approximate the following logarithms to four decimal places. Do not use a calculator.

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1. Suppose that is a number with .
   1. What is the value of ?
   2. Which of the following statements is true? Explain how you know.

***Because , is greater than and less than . Thus, (iii) is true. In fact, from the table above, we can narrow the value of down to between and because   
, and .***

Problem Set Sample Solutions

These problems should be solved without a calculator.

1. Complete the following table of logarithms without using a calculator; then, answer the questions that follow.

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* 1. What is ? How does that follow from the definition of a base logarithm?

***Since , we know that .***

* 1. What is for an integer ? How does that follow from the definition of a base logarithm?

***By the definition of the logarithm, we know that .***

* 1. What happens to the value of as gets really large?

***For any , there exists so that . As gets really large, gets large. Since   
, as gets large, gets large.***

* 1. For , what happens to the value of as gets really close to zero?

***For any , there exists so that . Then . As gets closer to zero, gets larger. Thus, is negative, and gets large as the positive number gets close to zero.***

1. Use the table of logarithms below to estimate the values of the logarithms in parts (a)–(h).

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1. If , find the value of .

1. If is a positive integer and , how many digits are there in ? Explain how you know.

***Since , we know ; therefore, has digits.***

1. If is a positive integer and , how many digits are there in ? Explain how you know.

***Since , we know ; therefore, has digits.***

1. Vivian says , while her sister Lillian says that . Which sister is correct? Explain how you know.

***Both sisters are correct. Since , we can write . However, we could also write , so . Both calculations give .***

1. Write the logarithm base of each number in the form , where is the exponent from the scientific notation, and is a positive real number.
2. For each of the following statements, write the number in scientific notation, and then write the logarithm base 10 of that number in the form , where is the exponent from the scientific notation, and is a positive real number.
   1. The speed of sound is .

***,*** ***so***

* 1. The distance from Earth to the Sun is million miles.

***,*** ***so*** **.**

* 1. The speed of light is .

***, so*** **.**

* 1. The weight of the earth is .

***so .***

* 1. The diameter of the nucleus of a hydrogen atom is .

***,*** ***so*** **.**

* 1. For each part (a)–(e), you have written each logarithm in the form , for integers and positive real numbers . Use a calculator to find the values of the expressions . Why are all of these values between and ?

These values are all between and because is between and . We can rewrite as   
. If we write for some exponent , then , so . This exponent is the base logarithm of .