Lesson 10: Building Logarithmic Tables

Classwork

Opening Exercise

Find the value of the following expressions without using a calculator.

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Formulate a rule based on your results above: If is an integer, then .

Example 1

100,000,000,000,000 Miles

1,000,000 times farther away

Exercises

1. Find two consecutive powers of so that is between them. That is, find an integer exponent so that
2. From your result in Exercise 1, is between which two integers?
3. Find a number to one decimal place so that , and use that to find under and over estimates for .
4. Find a number to two decimal places so that , and use that to find under and over estimates for .
5. Repeat this process to approximate the value of to decimal places.
6. Verify your result on your calculator, using the log button.
7. Use your calculator to complete the following table. Round the logarithms to decimal places.

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1. What pattern(s) can you see in the table from Exercise 7 as is multiplied by ? Write the pattern(s) using logarithmic notation.
2. What pattern would you expect to find for ? Make a conjecture and test it to see whether or not it appears to be valid.
3. Use your results from Exercises and to make a conjecture about the value of for any positive
integer .
4. Use your calculator to complete the following table. Round the logarithms to decimal places.

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1. What pattern(s) can you see in the table from Exercise 11? Write them using logarithmic notation.
2. What pattern would you expect to find for ? Make a conjecture and test it to see whether or not it appears to be valid.
3. Combine your results from Exercises and to make a conjecture about the value of the logarithm for a multiple of a power of that is, find a formula for for any integer .

Lesson Summary

* The notation is used to represent .
* For integers , .
* For integers and , .
* For integers k and positive real numbers .

Problem Set

1. Complete the following table of logarithms without using a calculator; then, answer the questions that follow.

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* 1. What is ? How does that follow from the definition of a base-logarithm?
	2. What is for an integer ? How does that follow from the definition of a base- logarithm?
	3. What happens to the value of as gets really large?
	4. For , what happens to the value of as gets really close to zero?
1. Use the table of logarithms below to estimate the values of the logarithms in parts (a)–(h).

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* 1.
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	7.
	8.
1. If , find the value of .
2. If is a positive integer and , how many digits are there in ? Explain how you know.
3. If is a positive integer and , how many digits are there in ? Explain how you know.
4. Vivian says , while her sister Lillian says that Which sister is correct? Explain how you know.
5. Write the logarithm base of each number in the form , where is the exponent from the scientific notation, and is a positive real number.
	1.
6. For each of the following statements, write the number in scientific notation and then write the logarithm base 10 of that number in the form , where is the exponent from the scientific notation, and is a positive real number.
	1. The speed of sound is .
	2. The distance from Earth to the Sun is million miles.
	3. The speed of light is .
	4. The weight of the earth is .
	5. The diameter of the nucleus of a hydrogen atom is .
	6. For each part (a)–(e), you have written each logarithm in the form , for integers and positive real numbers . Use a calculator to find the values of the expressions Why are all of these values between and ?